2016 NSW BOS Mathematics Exam Solutions

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Section I

1	2	3	4	5	6	7	8	9	10
B	C	B	A	B	A	A	D	C	D

Q1 $\sin\theta$ is a positive value and $\cos\theta$ is a negative value — B

Q2 $6\times\frac{1}{30}=\frac{1}{5}$ — C

Q3 Inverted, turning point at $(2,3)$ — B

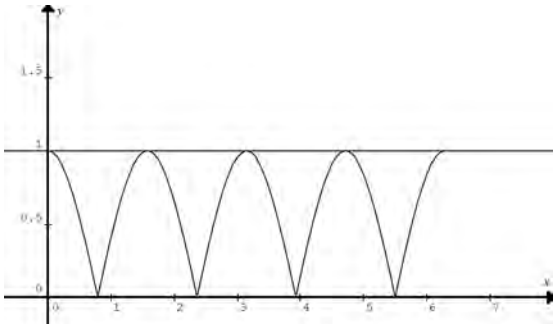
Q4 — A

Q5 $\frac{d}{dx}\ln(\cos x)=\frac{1}{\cos x}\times(-\sin x)=-\tan x$ — B

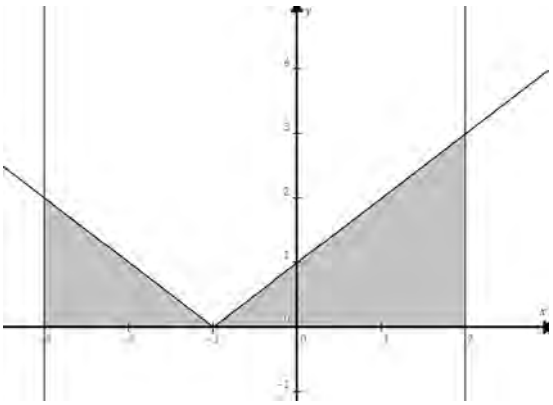
Q6 Period $=\frac{\pi}{3}$ — A

Q7 Area of shaded region $=\frac{7}{2\pi\times5}\times\pi\times5^2=\frac{35}{2}$ — A

Q8 — D



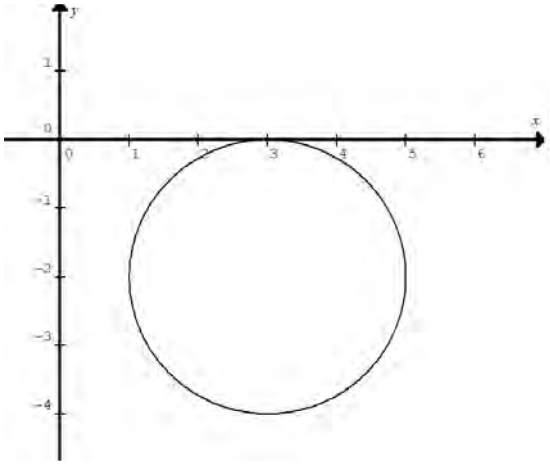
Q9 $\frac{1}{2}(2\times2+3\times3)=\frac{13}{2}$ — C



Q10 $4+\log_2 x=\log_2 2^4+\log_2 x=\log_2(16x)$ — D

Section II

Q11a



Q11b $\frac{d}{dx}\left(\frac{x+2}{3x-4}\right)=\frac{(3x-4)-3(x+2)}{(3x-4)^2}=\frac{-10}{(3x-4)^2}$

Q11c $|x-2|\le3$, $\sqrt{(x-2)^2}\le3$, $(x-2)^2\le9$, $(x-2)^2-3^2\le0$, $(x-2+3)(x-2-3)\le0$, $(x+1)(x-5)\le0$, $\therefore -1\le x\le5$

Q11d $\int_0^1(2x+1)^3\,dx=\left[\frac{(2x+1)^4}{4\times2}\right]_0^1=\frac{81}{8}-\frac{1}{8}=10$

Q11e $y=-5-4x$, $y=3-2x-x^2$

Let $-5-4x=3-2x-x^2$, $x^2-2x-8=0$, $(x-4)(x+2)=0$

$x=-2,4$ $\therefore$ $y=3,-21$ respectively

Points of intersection are $(-2,3)$ and $(4,-21)$.

Q11f $y=\tan x$, $\frac{dy}{dx}=\sec^2 x$

At $x=\frac{\pi}{8}$, gradient $=\frac{dy}{dx}=\sec^2\left(\frac{\pi}{8}\right)=\frac{1}{\left(\cos\frac{\pi}{8}\right)^2}\approx1.17$

Q11g $\sin\frac{x}{2}=\frac{1}{2}$, $\frac{x}{2}=\frac{\pi}{6},\frac{5\pi}{6}$, $x=\frac{\pi}{3},\frac{5\pi}{3}$

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Q12ai $\dfrac{y-1}{x-6}=\dfrac{4-1}{2-6}$, $-4(y-1)=3(x-6)$, $-4y+4=3x-18$
$\therefore\ 3x+4y-22=0$

Q12aii Length of $AD=\dfrac{|3\times1+4\times0-22|}{\sqrt{3^2+4^2}}=\dfrac{19}{5}$

Q12aiii Length of $BC=\sqrt{(1-4)^2+(6-2)^2}=5$

Area $=\dfrac{1}{2}\times5\times\dfrac{19}{5}=\dfrac{19}{2}$ unit^2

Q12bi $\angle BOA=x°$ (isosceles triangle)
$\angle CBO=2x°$ (external angle equals the sum of interior angles)

Q12bii $\angle BCO=2x°$ (isosceles triangle)
$\angle COB=180°-4x°$ (sum of angles)
$\therefore\ x°+180°-4x°+87°=180°$ (sum of angles), $\therefore\ x=\dfrac{87}{3}=29$

Q12c The cosine rule: $20^2+15^2-2\times20\times15\cos(90°-\theta°)=8^2$

$\cos(90°-\theta°)=\dfrac{20^2+15^2-8^2}{600}=0.935$, $90°-\theta°\approx21°$, $\theta\approx69$

Q12di $\dfrac{dy}{dx}=e^{3x}+3xe^{3x}=e^{3x}(1+3x)$

Q12dii

$$\int_0^2 e^{3x}(3+9x)\,dx=3\times\int_0^2 e^{3x}(1+3x)\,dx=3\times\left[xe^{3x}\right]_0^2=3\times2e^6=6e^6$$

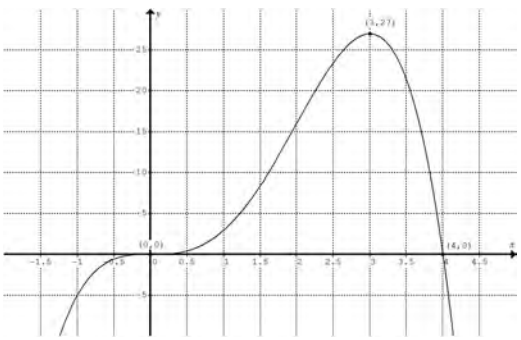
Q13ai $y=4x^3-x^4$, $\dfrac{dy}{dx}=12x^2-4x^3=4x^2(3-x)$

To find the stationary points, let $\dfrac{dy}{dx}=0$, $\therefore\ x=0$ and $y=0$ or $x=3$ and $y=27$.

x	<0	0	$0<x<3$	3	>3
dy/dx	positive	0	positive	0	negative
nature		inflection		max. t. p.	

Stationary point $(0,0)$ is a point of inflection, and stationary point $(3,27)$ is a local maximum.

Q13aii $y=4x^3-x^4=x^3(4-x)$, x-intercepts: $x=0$, $x=4$



Q13bi $x^2-4x+4=12y+12$, $(x-2)^2=4(3)(y+1)$, focal length $=3$

Q13bii Vertex $(2,-1)$, focus $(2,2)$

Q13ci $A=10$

Q13cii $5=10e^{-163k}$, $e^{163k}=2$, $k=\dfrac{\ln(2)}{163}\approx0.00425$

Q13d Shaded area $=\displaystyle\int_0^1\left(\sqrt{2}\cos\frac{\pi x}{4}-x\right)dx=\left[\frac{4\sqrt{2}}{\pi}\sin\frac{\pi x}{4}-\frac{x^2}{2}\right]_0^1$

$=\dfrac{4\sqrt{2}}{\pi}\sin\dfrac{\pi}{4}-\dfrac{1}{2}=\dfrac{4}{\pi}-\dfrac{1}{2}$

Q14ai Approximate increase in area
$=2\times\dfrac{2-0}{6}[(3-2.5)+4(2.78-2.38)+0]=1.4$ m^2

Q14bi $A_1=0.65\times100000+5000$
$A_2=0.65A_1+5000=0.65(0.65\times100000+5000)+5000$

Q14bii $A_2=0.65^2\times100000+5000+0.65\times5000$
$A_3=0.65A_2+5000=0.65^3\times100000+5000+0.65\times5000+0.65^2\times5000$
$A_n=0.65^n\times100000+S_n=0.65^n\times100000+\dfrac{5000(1-0.65^n)}{0.35}$

Q14biii $A_{14}=0.65^{14}\times100000+\dfrac{5000(1-0.65^{14})}{0.35}\approx14500$

Q14ci Let w m be the length of the enclosure. $wx=720$, $\therefore\ w=\dfrac{720}{x}$
$\therefore\ \ell=5x+w=5x+\dfrac{720}{x}$

Q14cii Let $\dfrac{d\ell}{dx}=0$, $5-\dfrac{720}{x^2}=0$, $x^2=144$, $x=12$

When $x<12$, $\dfrac{d\ell}{dx}<0$; when $x>12$, $\dfrac{d\ell}{dx}>0$, $\therefore$ ℓ is a minimum when $x=12$.

Minimum $\ell=5\times12+\dfrac{720}{12}=120$

Q14d $1+x+x^2+x^3+x^4=\dfrac{1(x^5-1)}{x-1}$

$\therefore\ \lim\limits_{x\to1}\dfrac{x^5-1}{x-1}=\lim\limits_{x\to1}(1+x+x^2+x^3+x^4)=5$

Q14e $\log2+\log4+\log8+\cdots+\log512$
$=\log2+\log2^2+\log2^3+\cdots+\log2^9$
$=\log2+2\log2+3\log2+\cdots+9\log2=45\log2$

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Q15a For C_1, volume $= \frac{1}{2}\times\frac{4}{3}\pi\times 2^3 = \frac{16\pi}{3}$

C_2 is the reflection of C_1 in the y-axis and the dilation of C_1 by a factor of $\frac{3}{2}$ in the positive x-direction.

For C_2, volume $= \frac{3}{2}\left(\frac{1}{2}\times\frac{4}{3}\pi\times 2^3\right) = 8\pi$

Volume of the solid of revolution $= \frac{16\pi}{3} + 8\pi = \frac{40\pi}{3}$ unit^3

Q15bi $\Pr(8) + \Pr(8'8) + \Pr(8'8'8) = \frac{1}{8} + \frac{7}{8}\times\frac{1}{8} + \frac{7}{8}\times\frac{7}{8}\times\frac{1}{8}$

$= \frac{1}{8} + \frac{7}{8}\times\frac{1}{8} + \left(\frac{7}{8}\right)^2\times\frac{1}{8}$

Q15bii $\frac{1}{8} + \frac{7}{8}\times\frac{1}{8} + \left(\frac{7}{8}\right)^2\times\frac{1}{8} + \cdots + \left(\frac{7}{8}\right)^{n-2}\times\frac{1}{8} > \frac{3}{4}$

$\frac{\frac{1}{8}\left(1-\left(\frac{7}{8}\right)^{n-1}\right)}{1-\frac{7}{8}} > \frac{3}{4}$, $1-\left(\frac{7}{8}\right)^{n-1} > \frac{3}{4}$, $\left(\frac{7}{8}\right)^{n-1} < \frac{1}{4}$

$\ln\left(\frac{7}{8}\right)^{n-1} < \ln\frac{1}{4}$, $(n-1)\ln\left(\frac{7}{8}\right) < \ln\frac{1}{4}$, $n-1 > \frac{\ln\frac{1}{4}}{\ln\frac{7}{8}}$

$\therefore\ n-1 > \frac{\ln\frac{1}{4}}{\ln\frac{7}{8}}$, $n-1 > 10.4$, $n > 11.4$

∴ smallest n value is 12.

Q15ci $\angle FCB = \angle BAT = 90°$
$\angle CFB = \angle ABT$ (alternate angles are equal)
∴ the the third pair of corresponding angles are equal
∴ ΔFCB and ΔBAT are similar

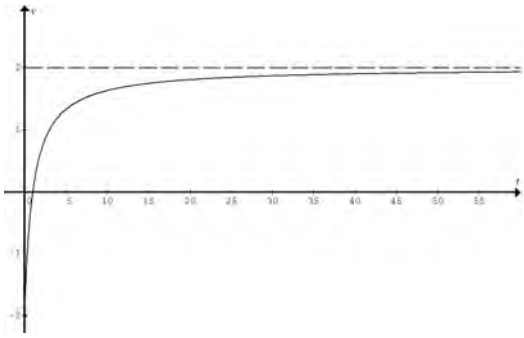
Q15cii $\angle TSA = \angle AEB = 90°$
$\angle TAS + \angle BAE = \angle ABE + \angle BAE = 90°$, $\therefore\ \angle TAS = \angle ABE$
∴ the the third pair of corresponding angles are equal
∴ ΔTSA and ΔAEB are similar

Q15ciii $\frac{h}{TA} = \frac{y}{AB}$ and $\frac{1}{x} = \frac{TA}{AB}$, $\therefore\ h = \frac{yTA}{AB}$, $\therefore\ h = \frac{y}{x}$

Q16ai $v = 2 - \frac{4}{t+1}$, when $t = 0$, $v = -2$

Q16aii $a = \frac{dv}{dt} = \frac{4}{(t+1)^2}$. When $v = 0$, $t = 1$, $a = 1$.

Q16aiii As $t \to \infty$, $v \to 2$



Q16aiv Distance $= \left|\int_0^1\left(2-\frac{4}{t+1}\right)dt\right| + \int_1^7\left(2-\frac{4}{t+1}\right)dt$

$= -\left[2t - 4\ln(t+1)\right]_0^1 + \left[2t - 4\ln(t+1)\right]_1^7$
$= -2 + 4\ln 2 + 14 - 4\ln 8 - 2 + 4\ln 2$
$= 10 - 4\ln 2$ metres

Q16bi $y = \frac{200}{1+19e^{-0.5t}}$

$\frac{dy}{dt} = -\frac{200}{\left(1+19e^{-0.5t}\right)^2}\times(-0.5)\times 19e^{-0.5t} = \frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^2}$

Q16bii When $t = 0$, $y = 10$, as $t \to \infty$, $y \to 200$, $\therefore\ 10 \le y < 200$

Q16biii $y = \frac{200}{1+19e^{-0.5t}}$, $19e^{-0.5t} = \frac{200}{y} - 1 = \frac{200-y}{y}$

$\frac{dy}{dt} = \frac{1900e^{-0.5t}}{\left(1+19e^{-0.5t}\right)^2} = \frac{100\left(\frac{200-y}{y}\right)}{\left(\frac{200}{y}\right)^2} = \frac{100y(200-y)}{200\times 200} = \frac{y}{400}(200-y)$

Q16biv Rate of growth $= \frac{y}{400}(200-y)$, an inverted parabola with axis of symmetry $y = -\frac{b}{2a} = 100$
∴ rate of growth is fastest when $y = 100$ yabbies

Please inform mathline@itute.com re conceptual and/or mathematical errors.

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