2011 NSW BOS Mathematics Exam Solutions

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Q1a $\sqrt[3]{\dfrac{651}{(4\pi)}} \approx 3.728$ by calculator.

Q1b $\dfrac{n^2-25}{n-5} = \dfrac{(n-5)(n+5)}{n-5} = n+5$

Q1c $2^{2x+1} = 32$, $2^{2x+1} = 2^5$, $2x+1=5$, $x=2$

Q1d $\dfrac{d}{dx}(\ln(5x+2)) = \dfrac{5}{5x+2}$

Q1e $2-3x \le 8$, $-3x \le 6$, $x \ge \dfrac{6}{-3}$, $x \ge -2$

Q1f

$$\frac{4}{\sqrt{5}-\sqrt{3}} = \frac{4}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{4(\sqrt{5}+\sqrt{3})}{2} = 2(\sqrt{5}+\sqrt{3})$$

Q1g $800 \times 0.02 = 16$ defective items

Q2ai $\alpha + \beta = 6$

Q2aii $\alpha\beta = 2$

Q2aiii $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{6}{2} = 3$

Q2b $2\sin x = -\sqrt{3}$ and $0 \le x \le 2\pi$, $\sin x = -\dfrac{\sqrt{3}}{2}$

$x = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

Q2c $y = (2x+1)^4$, $\dfrac{dy}{dx} = 8(2x+1)^3$

At $x=-1$, $y=1$, $\dfrac{dy}{dx} = -8$

Tangent: $y-1 = -8(x+1)$, $y = -8x-7$

Q2d $y = x^2e^x$, $\dfrac{dy}{dx} = 2xe^x + x^2e^x$ by the product rule

Q2e $\displaystyle\int \frac{1}{3x^2}\,dx = -\frac{1}{3x} + c$

Q3ai $t_1 = 3$, $d = 0.5$, $t_{25} = 3 + 24 \times 0.5 = 15$, \$15 million

Q3aii $S_{110} = \dfrac{110}{2}(2\times 3 + 109 \times 0.5) = 3327.5$, \$3327.5 million

Q3b The vertex of the parabola is equidistant from its focus $(3,2)$ and directrix $y=-4$, ∴ the vertex is $(3,-1)$.

Q3ci $\ell_1 : 3x+4y-12=0$

Let $x=0$, $y=3$, ∴ $B(0,3)$

Q3cii $m_1 = -\dfrac{3}{4}$, $m_2 = \dfrac{4}{3}$, $m_1m_2 = -1$, ∴ ℓ_1 and ℓ_2 are $\perp$

Q3ciii $\ell_1 : 3x+4y-12=0$. Let $y=0$, $x=4$, ∴ $A(4,0)$

∴ $AB = \sqrt{3^2+4^2} = 5$

ΔOAB and ΔOEB, ∴ $\dfrac{OE}{OA} = \dfrac{OB}{AB}$, $\dfrac{OE}{4} = \dfrac{3}{5}$, $OE = \dfrac{12}{5}$

Q3civ $\dfrac{BE}{BO} = \dfrac{OE}{OA}$, $\dfrac{BE}{3} = \dfrac{\frac{12}{5}}{4}$, $BE = \dfrac{9}{5}$

Q3cv *Area of* $\Delta BOE = \dfrac{1}{2} \times \dfrac{12}{5} \times \dfrac{9}{5} = \dfrac{54}{25}$ unit squares

Q4a $\dfrac{d}{dx}\left(\dfrac{x}{\sin x}\right) = \dfrac{\sin x - x\cos x}{\sin^2 x}$

Q4b $\displaystyle\int_e^{e^3} \frac{5}{x}\,dx = [5\ln x]_e^{e^3} = 5\ln e^3 - 5\ln e = 15 - 5 = 10$

Q4c $\dfrac{dy}{dx} = 6x-2$, $y = \displaystyle\int (6x-2)dx = 3x^2 - 2x + c$

$(-1,4)$, $4 = 3(-1)^2 - 2(-1) + c$, $c = -1$

∴ $y = 3x^2 - 2x - 1$

Q4di $y = \sqrt{9-x^2}$, $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{9-x^2}} \times (-2x) = -\dfrac{x}{\sqrt{9-x^2}}$

Q4dii $-\dfrac{x}{\sqrt{9-x^2}} = \dfrac{dy}{dx}$, $\dfrac{6x}{\sqrt{9-x^2}} = -6\dfrac{dy}{dx}$

$$\int \frac{6x}{\sqrt{9-x^2}}\,dx = -6\int \frac{dy}{dx}\,dx = -6\int dy = -6y + c = -6\sqrt{9-x^2} + c$$

Q4e $y \le 4 - x^2$ and $y \ge |x| - 2$

Q5ai $t_1 = a = 27$, $r = 2$, $t_{12} = ar^{12-1} = 27 \times 2^{11} = 55296$

Q5aii $t_n = 27 \times 2^{n-1} > 10000000$, $2^{n-1} > \dfrac{10000000}{27}$,
$n > 19.5$, $\therefore\ n = 20$, i.e. Day 20.

Q5aiii $S_{12} = \dfrac{27\left(2^{12}-1\right)}{2-1} = 110565$,
$earnings = \$0.005 \times 110565 \approx \$553$

Q5bi $\Pr(red_shirt_Monday) = \dfrac{3}{5}$

Q5bii $\Pr(red_red_red) = \dfrac{3}{5} \times \dfrac{2}{4} \times \dfrac{1}{3} = \dfrac{1}{10}$

Q5biii $\Pr(red_yellow_red) + \Pr(yellow_red_yellow)$
$= \dfrac{3}{5} \times \dfrac{2}{4} \times \dfrac{2}{3} + \dfrac{2}{5} \times \dfrac{3}{4} \times \dfrac{1}{3} = \dfrac{3}{10}$

Q5c $\displaystyle\int_0^{20} v\,dt \approx \frac{5}{3}\left(173 + 4 \times 81 + 2 \times 127 + 4 \times 195 + 168\right) = \frac{8495}{3}$ m

Q6ai $\angle CDE = \dfrac{3 \times 180°}{5} = 108°$

Q6aii $\angle CED = \dfrac{180° - 108°}{2} = 36°$
$\angle EPC = 360° - 3 \times 108° = 36°$
$\therefore\ \Delta EPC$ is isosceles

Q6b $\overline{PA}^2 + \overline{PB}^2 = 40$, $(x+1)^2 + y^2 + (x-3)^2 + y^2 = 40$,
$2x^2 - 4x + 10 + 2y^2 = 40$, $x^2 - 2x + 5 + y^2 = 20$,
$x^2 - 2x + 1 + y^2 = 16$, $(x-1)^2 + y^2 = 4^2$
$\therefore$ the locus is a circle of radius 4 centred at $(1,0)$.

Q6ci $P(0,2)$

Q6cii $\displaystyle\int_0^{\frac{\pi}{2}} 2\cos x\,dx = \left[2\sin x\right]_0^{\frac{\pi}{2}} = 2\sin\frac{\pi}{2} - 2\sin 0 = 2$

Q6ciii C

Q6civ $Area = 2 \times 4 = 8$ unit squares

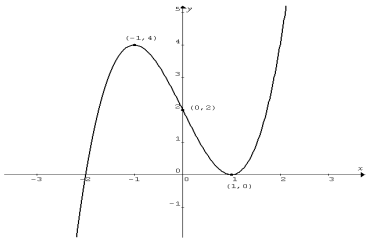
Q6cv $\displaystyle\int_{\frac{\pi}{2}}^{2\pi} 2\cos x\,dx = -2 - 2 + 2 = -2$

Q7ai $f(x) = x^3 - 3x + 2$, $f'(x) = 3x^2 - 3 = 3(x+1)(x-1)$
$f'(x) = 0$ at $x = -1$, $y = 4$ or $x = 1$, $y = 0$
Stationary points are $(-1,4)$ and $(1,0)$
Since the coefficient of the x^3 term is a positive value, the shape of $f(x)$ is $\frown\!\!\!\smile$.
OR

x	-2	-1	0	1	2
$f'(x)$	+	0	−	0	+

$\therefore\ (-1,4)$ is a local maximum and $(1,0)$ is a local minimum.

Q7aii



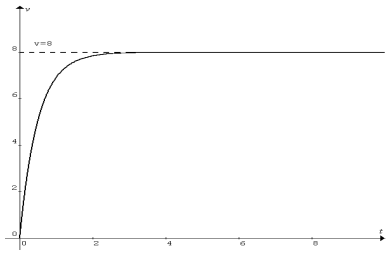
Q7bi $\dot{x} = 8 - 8e^{-2t}$
When $t = 0$, $v = \dot{x} = 8 - 8e^0 = 0$, $\therefore$ the particle is initially at rest.

Q7bii $a = \ddot{x} = 16e^{-2t} > 0$, $\therefore$ the acceleration is always positive.

Q7biii The particle starts from rest and its acceleration is always positive, $\therefore$ its velocity is positive for all $t > 0$, i.e. the particle moves in the positive direction for all $t > 0$.

Q7biv $v = \dot{x} = 8 - 8e^{-2t} \to 8$ as $t \to \infty$. The value of the constant is 8.

Q7bv



Q8ai $x^2 + 20^2 - 2(x)(20)\cos 60° = 22^2$
$x^2 + 400 - 20x = 484$, $\therefore\ x^2 - 20x - 84 = 0$

Q8aii $x = \dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-84)}}{2} = \dfrac{20 \pm \sqrt{736}}{2}$

$\therefore\ PL = \dfrac{20 + \sqrt{736}}{2} \approx 24$ km

Q8bi $\displaystyle V = \int_0^h \pi x^2\,dy = \int_0^h \pi y\,dy = \left[\frac{\pi y^2}{2}\right]_0^h = \frac{1}{2}\pi h^2$

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Q8bii $V_{cylinder} = \pi\left(\sqrt{h}\right)^2 h = \pi h^2$

$\therefore \dfrac{V_{paraboloid}}{V_{cylinder}} = \dfrac{\frac{1}{2}\pi h^2}{\pi h^2} = \dfrac{1}{2}$

Q8ci $R = 1 + \dfrac{6}{12\times 100} = 1.005$

End of nth month	$\$P_n$
1	100×1.005^1
2	$100\times 1.005^1 + 100\times 1.005^2$
3	$100\times 1.005^1 + 100\times 1.005^2 + 100\times 1.005^3$
…	…
420	$\dfrac{100\times 1.005\left(1.005^{420}-1\right)}{0.005} \approx 143183$

Q8cii1

End of nth month	$\$A_n$
1	$29227\times 1.005 + M\times 1.005$
2	$\left(29227\times 1.005 + M + M\times 1.005\right)\times 1.005$ $= 29227\times 1.005^2 + M\left(1.005 + 1.005^2\right)$

Q8cii2 At the end of 240th month after starting the new contributions, A_{240}

$= 29227\times 1.005^{240} + M\left(1.005 + 1.005^2 + ... + 1.005^{240}\right) = 800000$

$\therefore M = \dfrac{800000 - 29227\times 1.005^{240}}{1.005 + 1.005^2 + ... + 1.005^{240}} = \dfrac{703252.6538}{464.3511} = 1514.48$

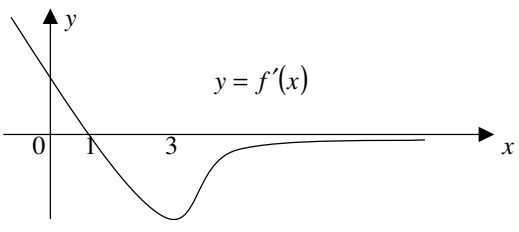
Q9ai ΔABC and ΔADE are similar because $\dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{BC}{DE} = \dfrac{1}{2}$.

Q9aii BC and DE are parallel, $\therefore \angle BCD = \angle CDE$ and $\angle CBE = \angle BED$, $\therefore$ ΔFBC and ΔFED are similar.
Since $BC : DE = 1 : 2$, $\therefore BF : FE = 1 : 2$

Q9bi $\left(2 + \dfrac{t^2}{t+1}\right) - \left(1 + \dfrac{1}{t+1}\right) = 1 + \dfrac{t^2-1}{t+1} = \dfrac{t^2+t}{t+1} = \dfrac{t(t+1)}{t+1} = t$

Q9bii $\Delta V = \displaystyle\int_0^4 t\,dt = \left[\frac{t^2}{2}\right]_0^4 = 8$ litres

Q9c



Q9di $\dfrac{1}{\sqrt{n+1}+\sqrt{n}} = \dfrac{1}{\sqrt{n+1}+\sqrt{n}} \times \dfrac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}$

$= \dfrac{\sqrt{n+1}-\sqrt{n}}{n+1-n} = \sqrt{n+1}-\sqrt{n}$

Q9dii $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + ... + \dfrac{1}{\sqrt{99}+\sqrt{100}}$

$= \sqrt{2} - \sqrt{1} + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + ... + \sqrt{100} - \sqrt{99}$

$= \sqrt{100} - \sqrt{1} = 9$

Q10ai $I = 10^{-12}\times e^{0.1L} = 10^{-12}\times e^{0.1\times 110} \approx 6.0\times 10^{-8}$ watt/m^2
(Comment: The correct formula is $I = 10^{-12}\times 10^{0.1L} = 10^{0.1L-12}$)

Q10aii $I = 10^{-12}\times e^{0.1L} = 8.1\times 10^{-9}$, $e^{0.1L} = 8.1\times 10^3$

$L = \dfrac{\ln\left(8.1\times 10^3\right)}{0.1} \approx 90$ decibels

Q10aiii $I = 10^{-12}\times e^{0.1L}$, $L = 10\ln\left(10^{12} I\right)$

Let $L_i = 10\ln\left(10^{12} I_i\right)$ and $L_f = 10\ln\left(10^{12} I_f\right)$

$\Delta L = L_f - L_i = 10\ln\left(10^{12} I_f\right) - 10\ln\left(10^{12} I_i\right) = 10\ln\dfrac{I_f}{I_i}$

If $I_f = 2I_i$, $\Delta L \approx 6.9$ decibels

Q10bi *Length of fencing P = arc length + 2 times radius*
$\therefore P = r\theta + 2r = r(\theta + 2)$

Q10bii $r\theta = P - 2r$

Sector area $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r(P-2r) = \dfrac{1}{2}P.r - r^2$

Q10biii Let $\dfrac{dA}{dr} = \dfrac{1}{2}P - 2r = 0$, $r = \dfrac{1}{4}P$ to maximise A.

Q10biv $P = r(\theta+2)$, $P = \dfrac{1}{4}P(\theta+2)$, $1 = \dfrac{1}{4}(\theta+2)$, $\theta = 2$

Q10bv Since $0 < \theta < 2\pi$, $0 < r\theta < 2\pi r$,
$0 + 2r < r\theta + 2r < 2\pi r + 2r$, $\therefore 2r < P < 2r(\pi+1)$

$\therefore r < \dfrac{P}{2}$ and $\dfrac{P}{2(\pi+1)} < r$

$\therefore \dfrac{P}{2(\pi+1)} < r < \dfrac{P}{2}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.

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