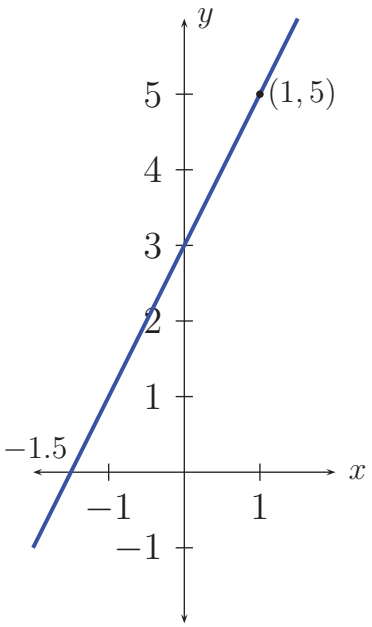
Q1

1a Straight Lines.

When $x = 0$, $y - 2(0) = 3 \implies y = 3$–this is the y-intercept.
When $y = 0$, $0 - 2x = 3 \implies x = -1.5$–this is the x-intercept.



It's good sketching etiquette to also plot one point on the graph–we chose $(1, 5)$, although this was not asked for here!

1b Solving Equations.

The first thing to notice is that $x \neq 0$, as the LHS of the equation is undefined at $x = 0$.

$$\begin{aligned} \frac{5x-4}{x} &= 2 \\ 5x - 4 &= 2x \\ 5x - 2x &= 4 \\ 3x &= 4 \\ x &= 4/3 = 1\frac{1}{3} \end{aligned}$$

1c Solving Equations with absolute value symbol.

$|\,x + 1\,| = 5 \implies$
$x + 1 = 5 \quad$ or $\quad x + 1 = -5$
$\implies x = -6, 4$

1d Tangent gradient.

$y = x^4 - 3x \implies y' = 4x^3 - 3$
When $x = 1, y' = 4.1^3 - 3 = 1$

1e Trig Equations.

$2\cos\theta = 1 \implies \cos\theta = 0.5 \implies \theta = \dfrac{\pi}{3}$
(using the exact triangle, or even a calculator!)

1f Equations with logs.

Theory says that the log form, $\log_a b = c$ can be written in index form as, $b = a^c$.

Hence,
$\ln x = 2 \implies \log_e x = 2 \implies x = e^2 = 7.3891$ (4 d.p.)

Q2

2a Differentiation.

2.a.i
$\dfrac{d}{dx}(x\sin x) = \sin x + x\cos x$ (using the product rule).

2.a.ii
$\dfrac{d}{dx}\left((e^x+1)^2\right) = 2\,(e^x+1)^{2-1} \times e^x$ (using the chain rule)
$= 2e^x\,(e^x + 1)$

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2b Integration.

2.b.i

$\int 5\,dx = 5x + c$ where c is a constant.

2.b.ii

$$\int \frac{3}{(x-6)^2}\,dx = \int 3(x-6)^{-2}\,dx$$
$$= 3\int (x-6)^{-2}\,dx = 3\frac{(x-6)^{-2+1}}{-2+1} + c$$
$$= \frac{-3}{x-6} + c \text{ where } c \text{ is a constant.}$$

2.b.iii

$$\begin{aligned} I &= \int_1^4 x^2 + x^{1/2}\,dx \\ &= \frac{x^3}{3} + \frac{2x^{3/2}}{3}\Big|_1^4 \\ &= \frac{4^3}{3} + \frac{2(4)^{3/2}}{3} - \frac{1^3}{3} - \frac{2(1)^{3/2}}{3} \\ &= 64/3 + 16/3 - 1/3 - 2/3 \\ &= 80/3 - 1 = 77/3 \\ &= \boxed{25\frac{2}{3}} \end{aligned}$$

2c Summation notation.

$$\sum_{k=1}^{4}(-1)^k\,k^2$$
$$= (-1)^1\,1^2 + (-1)^2\,2^2 + (-1)^3\,3^2 + (-1)^4\,4^2$$
$$= -1 + 4 - 9 + 16$$
$$= \boxed{10}$$

Q3

3a Arithmetic Series.

We have $n = 21$ (number of terms)
First term, $T_1 = 3$.
Twenty first (and last) term, $T_{21} = 53$.

Theory says $S = \frac{n}{2}(a + l)$ where n is the number of terms, a is the first term, l is the last term.

Therefore, $S_{21} = \frac{21}{2}(3 + 53) = \frac{21}{2}(56) = 21 \times 28 = \boxed{588}$.

3b Coordinate geometry and lines etc.

3.b.i

Using the point-point formula for a straight line we have,

$$\begin{aligned} \frac{y-1}{x-2} &= \frac{5-1}{5-2} \\ y - 1 &= \frac{4}{3}(x-2) \\ 3y - 3 &= 4x - 8 \\ 4x - 3y - 5 &= 0 \end{aligned}$$

3.b.ii

$$\begin{aligned} d &= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| \quad \text{Formula for shortest distance between point } (x_1, y_1) \text{ and line } ax + by + c = 0 \\ &= \left|\frac{4.1 - 3.3 - 5}{\sqrt{4^2+3^2}}\right| \\ &= \left|\frac{4 - 9 - 5}{5}\right| \\ &= \boxed{2} \end{aligned}$$

3.b.iii

The equation of a circle with radius r and centre (h, k) is given by $(x-h)^2+(y-k)^2 = r^2$ and here we have $r = 2$ (by part (ii)), and the centre is $(h, k) = (1, 3)$, so the circle is

$$(x-1)^2 + (y-3)^2 = 4$$

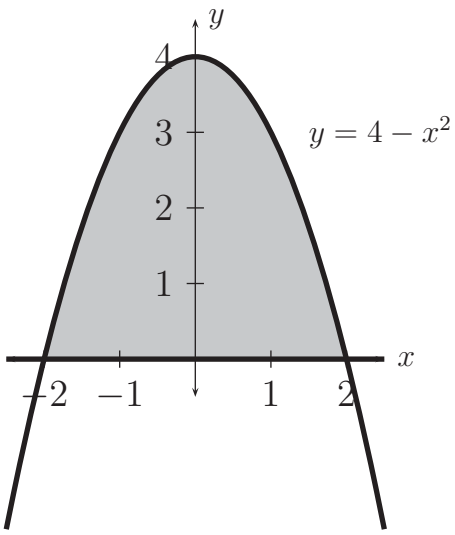
3c Shading a region.

To shade the region we need to firstly sketch the boundary curves; $y = 0$, $y = 4 - x^2$. The

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pick a point not on any boundary, say, $(0,1)$ and substitute it into the inequalities to see which side to shade in; if the inequality is satisfied then you shaed that side in, if not, then shade the other side in.
For $y \leq 4 - x^2 \implies 1 \leq 4 - o \implies 1 \leq 4$, this is true so shade under the curve $y = 4 - x^2$.
For $y \geq 0 \implies 1 \geq 0$, this is true so shade above the line $y = 0$.



3d Trapezoidal rule.

$A = \frac{50}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6]$
$= \frac{50}{3}[210 + 4(220) + 2(200) + 4(190) + 2(210) + 4(240) + 240]$
$= \boxed{64500m^2}$

Q4

4a Limiting Sum.

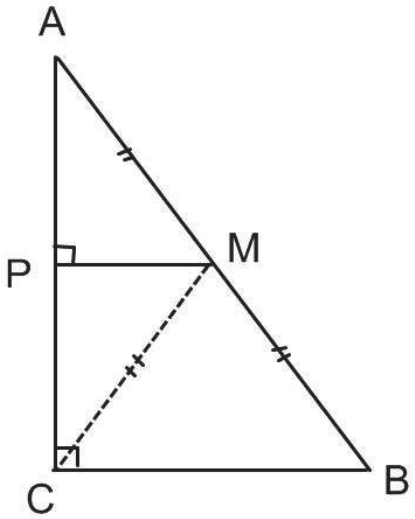
$S = 1.2 + 0.9 \times 1.2 + 0.9^2 \times 1.2 + \cdots = \frac{1.2}{1 - 0.9} = \boxed{12 \text{ m}}$

(The limiting sum exists as $r = 0.9 < 1$.)

4b The Discriminant.

The quadratic equation $x^2 - (k+4)x + (k+7) = 0$ has discriminant
$\Delta = b^2 - 4ac = (k+4)^2 - 4(k+7) = k^2 + 8k + 16 - 4k - 28 = k^2 + 4k - 12$.
For equal roots $\Delta = 0$ and hence we solve $k^2 + 4k - 12 = 0 \implies (k-2)(k+6) = 0 \implies k = -6, 2$.

4c



4.c.i
We show similarity by showing two corresponding angles are equal.
$\angle A$ is common
$\angle APM = \angle ACB = 90^o$ (Given)
$\therefore \Delta AMP \,|||\, \Delta ABC$ (AA–Two corresponding angles are equal.)

4.c.ii
The ratios of corresponding sides of similar triangles are equal.
So by (i) since ΔAMP and ΔABC are similar then we have the ratio

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$\frac{AM}{AB} = \frac{AP}{AC}$ and since M is the midpoint of AB then $AB = 2AM$ and hence,

$\frac{AP}{AC} = \frac{AM}{2AM} = \frac{1}{2} \implies$ ratio is $\boxed{1:2}$.

4.c.iii

We show that $\Delta PMC \equiv \Delta PMA$

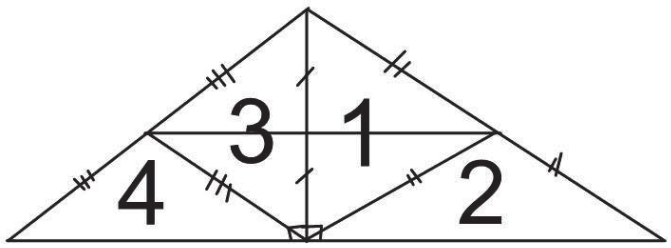
$$
\begin{array}{lrl}
(S) & PM & \text{is a common side} \\
 & \angle MPC & = 180 - \angle MPA \text{(straight angle)} \\
 & & = 180 - 90 \\
 & & = 90 \\
(A) & \text{So } \angle MPC & = \angle MPA \\
(S) & AP & = PC \text{ (since by part (ii) } AP : AC = 1:2, \text{ so } P \text{ is the midpoint.)} \\
 & \therefore \Delta PMC & \equiv \Delta PMA \text{ (SAS)} \\
 & \text{Thus } CM & = AM \text{ (Corresponding sides in comgruent triangles are equal)}
\end{array}
$$

Hence ΔAMC is isosceles as two sides are equal.

4.c.iv $CM = MB$ (since $CM = AM$ by above) so that ΔCMB is also an isosceles triangle.
So ΔABC comprises two isosceles triangles, ΔAMC and ΔCMB.

4.c.v

The idea here is to drop a perpendicular so that initially the triangle is split up into two right angled triangles, each of which comprises two isosceles triangles by drawing the line parallel to the base half way. Therefore the original triangle comprises four isosceles triangles, as shown in the diagram.



Q5

5a 5.a.i The line AB is: $y = \sqrt{3}x - 3$ and has gradient $\sqrt{3}$ and so the line BC, perpendicular to AB, has gradient $-1/\sqrt{3}$. (negative reciprocal).

Find B:
When $y = 0 \implies 0 = \sqrt{3}x - 3$

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$\implies x = \sqrt{3} \therefore B(\sqrt{3}, 0)$.

Therefore, using the point-gradient formula, line BC has equation:

$y - 0 = -\frac{1}{\sqrt{3}}(x - \sqrt{3}) \implies \boxed{y = -\frac{1}{\sqrt{3}}x + 1}$

5.a.ii In order to find the area we need to first find A and C.
Get A: When $x = 0$ on AB then $y = \sqrt{3}(0) - 3 = -3 \implies A(0, -3)$
Get C: When $x = 0$ on BC then $y = -\frac{1}{\sqrt{3}}(0) + 1 = 1 \implies C(0, 1)$
Thus the distance $AC = 1 + 3 = 4$ and therefore the area of ΔABC is

$$\text{Area} = 0.5 \times 4 \times \sqrt{3} = \boxed{2\sqrt{3}}$$

(Half the base times the height)

5b Probability.

5.b.i
As there are three car parks he has one chance in three of choosing correctly the first time, so probability $= \frac{1}{3}$.

5.b.ii
The probability of not choosing correctly the first time is $\frac{2}{3}$ and once on an incorrect floor, the subsequent probability that his next choice is also incorrect is reduced to $\frac{1}{2}$ (as there are only two unsearched floors left). Once on the next incorrect floor, he is left to search the final floor. Thus we have

Probability of searching all three floors
$= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

5.b.iii

$$\left(\frac{2}{3}\right)^5 = \frac{32}{243}$$

5c Geometry

5.c.i
Recall the area formula of a triangle given two sides and an included angle is $\frac{1}{2}ab\sin C$.

So area $A = 0.5 \times 2 \times 2 \times \sin\theta = 2\sin\theta$.
We solve $2\sin\theta = \sqrt{3} \implies \sin\theta = \sqrt{3}/2 \implies \theta = \pi/3$ or $\pi - \pi/3$ since the sine function is positive in quadrants 1 and 2. Thus $\theta = \frac{2\pi}{3}$ is the other value required.

5.c.ii
Q5.c.ii.1 Given that $\theta = \frac{\pi}{3}$ the area of the sector is
$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$
Q5.c.ii.2
Arc length AB is $l = r\theta = 2 \times \pi/3 = \frac{2\pi}{3}$
Since the angle at O is $\pi/3$ this means the triangle is equilateral and hence all sides are equal, and so $AB = 2$. Therefore the perimeter of the minor segment is $\boxed{\frac{2\pi}{3} + 2}$.

Q6

6a Volume of solid of revolution.

The volume is
$V = \pi\int_a^b y^2\,dx = \pi\int_{-\pi/3}^{\pi/3} \sec^2 x\,dx =$
$\pi\tan x\big|_{-\pi/3}^{\pi/3} = 2\pi\tan\pi/3 = \boxed{2\sqrt{3}\pi}$

6b Exponential growth/decay.

6.b.i We have $Q = Ae^{-kt}$.
Given the half-life is $T_{1/2} = 1600$ years we may write
$Ae^{-kT_{1/2}} = A/2 \implies e^{-1600k} = 1/2 \implies$

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$-1600k = \ln(1/2) = -\ln 2$
Thereofore $k = \dfrac{\ln 2}{1600} = 0.000433217$ (best not to round to much and keep the decimals for further calculations)

6.b.ii
This calculation follows along the same lines as the half-life calculation,
$\dfrac{A}{3} = Ae^{-kt} \implies e^{-kt} = 1/3$
$\implies -kt = \ln(1/3) = -\ln 3$
Therefore, $t = \dfrac{\ln 3}{k} = \dfrac{\ln 3}{\ln 2} \times 1600 =$ $\boxed{2536 \text{ years}}$.

6c Parabola.

6.c.i
We have $y = ax^2 + bx$ so that $y' = 2ax + b$.

Feeding the given slopes into the derivative gives;
when $x = 0$, $2a(0) + b = 1.2 \implies b = 1.2$
when $x = 30$, $2a(30) + b = -1.8 \implies a = \dfrac{-1.8 - 1.2}{60} = -\dfrac{1}{20}$

6.c.ii
Updating the parabola equation:

$$y = -0.05x^2 + 1.2x$$

We need the vertex, so as the axis of symmetry is
$x = -b/2a = -1.2/(2 \times -0.05) = 12$ and so $y = -0.05(12)^2 + 1.2(12) = 7.2$ and the vertex is $(12, 7.2)$.
At P, $x = 30$ and $y = -0.05(30)^2 + 1.2(30) = -9$
Therefore the distance between the vertex and the horizontal line through P is $9 + 7.2 = \boxed{16.2}$.

Q7

7a Velocity/acceleration.

$\ddot{x} = 8e^{-2t} + 3e^{-t}$. We are given that when $t = 0, v = -6$ ms^{-1}, $x = 5$ m.

7.a.i We get $\dot{x}$ and x by integrating and using the constraints to find the integration constants.

$$\begin{aligned}
\dot{x} &= -4e^{-2t} - 3e^{-t} + c_1 \\
-6 &= -4.1 - 3.1 + c_1 \\
c_1 &= -6 + 4 + 3 = 1 \\
\text{Updating } \dot{x} &= 1 - 4e^{-2t} - 3e^{-t} \\
x &= t + 2e^{-2t} + 3e^{-t} + c_2 \\
5 &= 0 + 2 + 3 + c_2 \\
c_2 &= 0 \\
\text{Updating } x &= t + 2e^{-2t} + 3e^{-t}
\end{aligned}$$

7.a.ii
The particle comes to rest when $\dot{x} = 0$ so we solve (by factorising) the equation;
$1 - 4e^{-2t} - 3e^{-t} = 0 \implies (4e^{-t} - 1)(e^{-t} + 1) = 0$
$\implies e^{-t} = 1/4$ or $e^{-t} = -1$ (the latter gives no solution as the exponential function is always positive)

Therefore $-t = -\ln 4 \implies t = \ln 4 = 1.39$ s (2 d.p.)

7.a.iii
From part (ii) the particle comes to rest when $e^{-t} = 1/4$ (and so $t = \ln 4$), so substitution gives
$x = t + 2e^{-2t} + 3e^{-t} = \ln 4 + 2(1/4)^2 + 3(1/4) = \ln 4 + 1/8 + 3/4 =$ $\boxed{\ln 4 + 7/8 = 2.26 \text{ m (2 d.p.)}}$.

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7b

We have, $h = 1 + 0.7 \sin \dfrac{\pi t}{6}, 0 \le t \le 12$.

7.b.i

Period is $= \dfrac{2\pi}{b}$ ('formula') $= \dfrac{2\pi}{\pi/6} = 12$ hours.

7.b.ii

At low tide the sine function is a minimum and so equals -1, and hence low tide is $1 + 0.7 \times -1 = \boxed{0.3 \text{ m}}$.

Find the time when this occurs: solve

$1 + 0.7 \sin \dfrac{\pi t}{6} = 0.3 \implies 0.7 \sin \dfrac{\pi t}{6} = -0.7$

$\implies \sin \dfrac{\pi t}{6} = -1 \implies \dfrac{\pi t}{6} = 3\pi/2 + 2\pi n$ $(n \in \mathbb{Z})$

So $t = 6/\pi(3\pi/2 + 2\pi n) = 9 + 2n \implies t = 9$ (choosing $n = 0$).

Thus the time is 5 am +9 hours = 2 : 00 pm.

7.b.iii

$1 + 0.7 \sin \dfrac{\pi t}{6} \ge 1.35 \implies \sin \dfrac{\pi t}{6} \ge 0.5$

Now, $\sin \frac{\pi t}{6} = 0.5$ when

$\pi t/6 = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6$

$\therefore\ t = 1, 5, 13, 17$ and since the last two values are too big then we deduce that $1 \le t \le 5$ and hence the time satisfies 5 am + 1 hour $\le t \le$ 5 am + 5 hours $\implies$ 6 am $\le$ time $\le$ 10 am.

Q8

8a Graphs and gradient function.

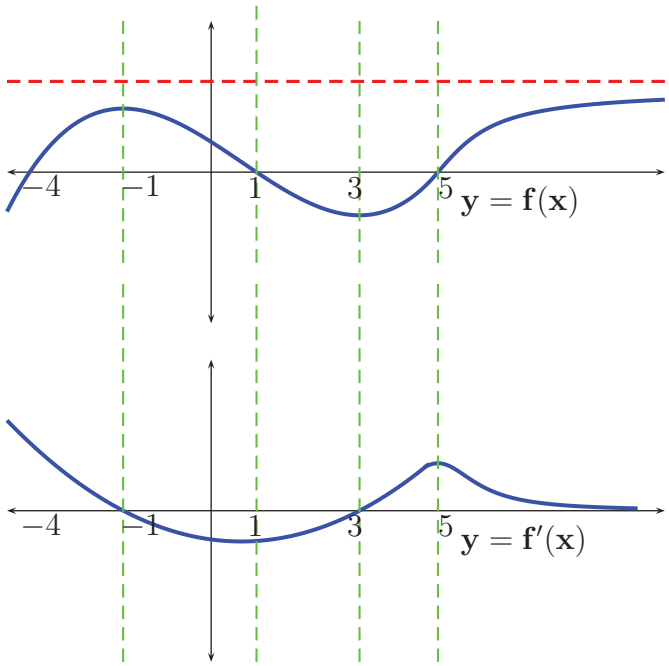
8.a.i The derivative is negative when going from left to right–the graph slopes downward and is decreasing.

This occurs when $-1 < x < 3$

8.a.ii The graphs flattens out as you go to the right, and the gradient approaches 0 (or at least it appears to from the graph).

So $x \to \infty$, $f'(x) \to 0^+$.

8.a.iii



The sketches are not to scale. The stationary points on y will correspond to points on the graph of y' that cross the x-axis (as stat points have $y' = 0$)

The inflexions on y will correspond to points on the graph of y' with $y'' = 0$.

Portions of the curve y with positive slope will correspond to portions of the graph of y' that are above the x-axis. Get it?

8b Financial applications of Series.

8.b.i

9% per annum means it is 9/12% = 0.75% per month.

At the end of the first month the amount owing after the repayment is,

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$A_1 = 350000 \times 1.0075 - 2937 = \$349\,688$

8.b.ii We use the sum of a geometric series with n terms to find the closed form of A_n;

$A_n = 346\,095 \times 1.005^n - M\left(1 + 1.005 + \cdots + 1.005^{n-1}\right)$

$$A_n = 346\,095 \times 1.005^n - M\frac{1.005^n - 1}{0.005}$$

Now, $A_{288} = 0$ if the loan is repaid in 288 months.
Solving for M we have,

$$M = \frac{346\,095 \times 1.005^{288} \times 0.005}{1.005^{288} - 1}$$
$$= \$2,270.31$$

8.b.iii
We use the same idea as the previous part but this time we substitute for M and solve for n;

$$\begin{aligned}
0 &= 346\,095 \times 1.005^n - 2937\left(\frac{1.005^n - 1}{0.005}\right) \\
0 &= 346\,095 \times 1.005^n \times 0.005 - 2937 \times 1.005^n + 2937 \\
1.005^n &= \frac{2937}{2937 - 346\,095 \times 0.005} \\
n \ln 1.005 &= \ln\left(\frac{2937}{2937 - 346\,095 \times 0.005}\right) \\
n &= \frac{\ln\left(\frac{2937}{2937 - 346\,095 \times 0.005}\right)}{\ln 1.005} \\
n &= 178.37
\end{aligned}$$

Hence it will take him 179 months or 14 years 11 months.

8.b.iv
The amount owing at the end of the 178^{th} month after the repayment for that month equals

$$A_{178} = 346\,095 \times 1.005^{178} - \frac{2937}{0.005}(1.005^{178} - 1) = \$1092.68$$

With interest, at the end of the 179^th month he still owes $1.005 \times 1092.68 = \$1098.15$
Total repaid is $178 \times 2937 + 1098.15 = \$523\,884.15$
Had he paid off with the lower new repayments he would have paid $288 \times 2270.31 = \$653\,849.28$.
He saved $\$653\,849.28 - \$523\,884.15 = \boxed{\$129\,965.13}$.

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Q9

9a Probability.

The probability of winning no prize at all in a week is $\frac{8}{9} \times \frac{15}{16} = \frac{15}{18} = \frac{5}{6}$.

Therefore the probability of winning no prizes three weeks in a row equals $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$.

Hence the probability that at least one of them wins a prize in three weeks is the complement of the probability that noone wins. Hence $1 - \frac{125}{216} = \frac{91}{216}$.

9b Maxima/minima.

9.b.i
\$1000 per km along shore and \$2600 underwater.
Therefore P to R is 5 km $\implies 5 \times \$1000 = \$5000$.
R to S is 3 km underwater $\implies 3 \times \$2600 = \$7800$.
Total Cost $= \$5000 + \$7800 = \$12\,800$.

9.b.ii Distance, $PS = \sqrt{3^2 + 5^2} = \sqrt{34}$.
Hence the cost is $\sqrt{34} \times \$2600 = \$15\,160.47$.

9.b.iii P to Q: $\$1000 \times (5 - x)$.
Q to S: $\$2600 \times \sqrt{x^2 + 3^2}$.

$C = 1000(5 - x + 2.6\sqrt{x^2 + 9})$

9.b.iv
We solve $\frac{dC}{dx} = 0$ to find the minimum cost and we show that at that value $\frac{d^2C}{dx^2} > 0$ (to verify it is a min).

$$\begin{aligned}
\frac{dC}{dx} &= 1000\left[-1 + \frac{2.6}{2}\left(x^2+9\right)^{-1/2} \times 2x\right] \\
\text{Put } \frac{dC}{dx} &= 0 \\
\frac{2.6x}{\sqrt{x^2+9}} &= 1 \\
2.6x &= \sqrt{x^2+9} \\
2.6^2x^2 &= x^2 + 9 \\
x^2(2.6^2 - 1) &= 9 \\
x &= \frac{3}{\sqrt{2.6^2-1}} \\
x &= 1.25
\end{aligned}$$

When $x = 1.25$ we have,

$\frac{d^2C}{dx^2} = 1000[2.6(x^2 + 9)^{-1/2} - \frac{2.6}{2}(x^2 + 9)^{-3/2} \times 2x^2]$
$= 1000(0.8 - 0.118343195) > 0$ and hence it is concave up and a minimum turning point.

Therefore the minimum cost is $C = 1000(5 - 1.25 + 2.6\sqrt{1.25^2 + 9}) = \$12\,200$.

9.b.v
Same method as in previous part, except there is a 1.1 in place of 2.6.

$C = 1000(5 - x + 1.1\sqrt{x^2 + 9})$

$$\begin{aligned}
\frac{dC}{dx} &= 1000\left[-1 + \frac{1.1}{2}\left(x^2+9\right)^{-1/2} \times 2x\right] \\
\text{Put } \frac{dC}{dx} &= 0 \\
\frac{1.1x}{\sqrt{x^2+9}} &= 1 \\
1.1x &= \sqrt{x^2+9} \\
1.1^2x^2 &= x^2 + 9 \\
x^2(1.1^2 - 1) &= 9 \\
x &= \frac{3}{\sqrt{1.1^2-1}} \\
x &= 6.546536 \text{ km}
\end{aligned}$$

But x must be less that 5 km! Hence choose

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$x = 5$ (on the 'boundary' of the domain),so that the cable is laid entirely underwater now.
$\therefore C = 1100 \times \sqrt{34} = \$6414.05$

Q10

Curve sketching.

We have $f(x) = x - x^2/2 + x^3/3$

10a

$f'(x) = 1 - x + x^2 = x^2 - x + 1.$

Stat points: Solve $f'(x) = 0 \implies$
$x^2 - x + 1 = 0.$
But $\Delta = b^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3 < 0$ and hence there are no solutions and no stationary points.

10b

$f''(x) = 2x - 1$. For possible inflexions we solve $f''(x) = 0 \implies 2x - 1 = 0 \implies x = 0.5$ and as $f''(0) = -1 < 0$ and $f''(1) = 1 > 0$ then the sign of f'' changes so this verifies that there is an inflexion when $x = 0.5$. The inflexion point is thus $\left(\frac{1}{2}, \frac{5}{12}\right)$.

10c

10.c.i

$$LHS = 1 - x + x^2 - \frac{1}{1+x}$$
$$= \frac{(1-x)(1+x) + x^2(1+x) - 1}{1+x}$$
$$= \frac{1 - x^2 + x^2 + x^3 - 1}{1+x}$$
$$= \frac{x^3}{1+x} = RHS$$

as required.

10.c.ii

$g(x) = \ln(x+1) \implies g'(x) = \frac{1}{x+1}$.

Also recall, $f'(x) = 1 - x + x^2$.

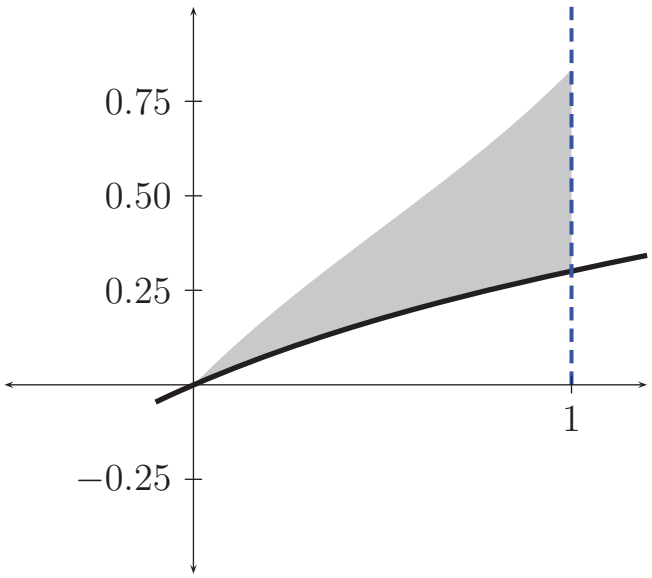
Clearly $\frac{x^3}{1+x} \geq 0$ for $x \geq 0$.

So by part (i),

$f'(x) - g'(x) = 1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x} \geq 0.$

Hence $f'(x) - g'(x) \geq 0 \implies f'(x) \geq g'(x)$ for $x \geq 0$ as required.

10d



10e

$LHS = \frac{1+x}{1+x} + 1.\ln(1+x) - 1 = \ln(1+x)$ as required.
We use this result in the next part.

10f

Area $= \int_0^1 x - x^2/2 + x^3/3 - \ln(1+x)\,dx$
$= x^2/2 - x^3/6 + x^4/12 - (x+1)\ln(x+1) + x + 1$ (using part (e))
$= 1/2 - 1/6 + 1/12 - 2\ln 2 + 2 - 1 = \boxed{0.03}$ (2 d.p.).

THE END.

Contact: Comments/corrections are appreciated, janh@hansendata.com.au,

Regards,

Jan Hansen 5 November 2009.

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