2005 NSW BOS Mathematics Exam Solutions

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Q1a 3.7

Q1b $x^3-27=x^3-3^3=(x-3)(x^2+3x+9)$

Q1c $4x+\tan x$

Q1d $\dfrac{(2x-3)}{2}-\dfrac{(x-1)}{5}=\dfrac{5(2x-3)-2(x-1)}{10}=\dfrac{8x-13}{10}$

Q1e $|x-3|\le 1,\ 2\le x\le 4$

Q1f $x^2=8(y-1)=4\times 2(y-1)$, vertex $(0,1)$, focus $(0,3)$

Q2a $\cos\theta=\dfrac{1}{\sqrt{2}},\ \theta=\dfrac{\pi}{4},\dfrac{7\pi}{4}$

Q2bi $\dfrac{d}{dx}(x\sin x)=x\cos x+\sin x$

Q2bii $\dfrac{d}{dx}\left(\dfrac{x^2}{x-1}\right)=\dfrac{2x(x-1)-x^2}{(x-1)^2}=\dfrac{x(x-2)}{(x-1)^2}$

Q2ci Let $u=x^3+1$, $\dfrac{du}{dx}=3x^2$, $\displaystyle\int\frac{6x^2}{x^3+1}dx$

$=\displaystyle\int\frac{2}{u}\frac{du}{dx}dx=\int\frac{2}{u}du=2\log_e u+C=2\log_e(x^3+1)+C$

Q2cii $\displaystyle\int_0^{\frac{\pi}{6}}\cos 3x dx=\left[\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{6}}=\frac{1}{3}\sin\frac{\pi}{2}-0=\frac{1}{3}$

Q2d $y=\log_e x$, $m_T=\dfrac{dy}{dx}=\dfrac{1}{x}=\dfrac{1}{e}$

Equation of tangent: $y-1=\dfrac{1}{e}(x-e)$, $\therefore y=\dfrac{1}{e}x$

Q3a $\displaystyle\sum_{n=3}^{5}(2n+1)=(2\times 3+1)+(2\times 4+1)+(2\times 5+1)=27$

Q3bi $\cos\theta=\dfrac{7^2+8^2-13^2}{2\times 7\times 8}$, $\theta=120^o$

Q3bii $Area=\dfrac{1}{2}\times 7\times 8\sin 120^o=24.2\text{ cm}^2$

Q3ci $m_{AD}=m_{BC}=\dfrac{6-0}{12-9}=2$,

Equation of line AD: $y-0=2(x-6)$, $y=2x-12$

Q3cii Solve $x-2y=0$ and $y=2x-12$ simultaneously to find the intersection D: $x-2(2x-12)=0$, $\therefore 3x=24$, $x=8$, $y=4$, D is $(8,4)$.

Q3ciii DE is 3 units long, E is $(11,4)$.

Q3civ $\angle CDE=\angle DOA$ (Corresponding angles)
$\angle DCE=\angle ODA$ (Corresponding angles)
$\therefore \angle DEC=\angle OAD$, $\therefore \Delta OAD$ and ΔDEC are similar.

Q3cv $AD:EC=OA:DE=6:3=2:1$
or $\dfrac{AD}{EC}=2$

Q4ai Length of arc $AB=r\theta=90\times 0.6=54$ cm

Q4aii Length of $AB=2\times 90\sin 0.3=53.2$ cm

Q4aiii Sector area $=\dfrac{1}{2}r^2\theta=\dfrac{1}{2}\times 90^2\times 0.6=2430\text{ cm}^2$

Q4bi $f(x)=(x+3)(x^2-9)=(x+3)(x+3)(x-3)=0$
$\therefore x=-3$ or 3

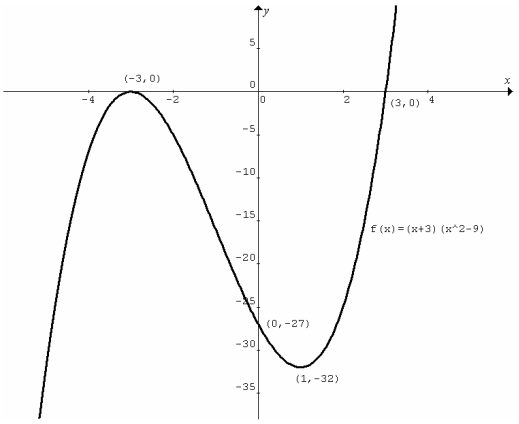
Q4bii $f'(x)=(x^2-9)+2x(x+3)=(x+3)(3x-3)$

Stationary points: $(x+3)(3x-3)=0$, $\therefore x=-3$ and $y=0$ or $x=1$ and $y=-32$

Nature of stationary points:

At $x=$	-4	-3	0	1	2
$f'(x)$	>0	$=0$	<0	$=0$	>0
Nature		Local max.		Local min.	

Q4biii



Q4biv $f'(x)=3x^2+6x-9$, $f''(x)=6x+6=0$, $x=-1$.
$f''(x)<0$ for $x<-1$, $\therefore f(x)$ is concave down for $x<-1$.

Q5a $\log_3 7=\dfrac{\log_{10}7}{\log_{10}3}=1.77$

Q5b $\angle ADC=180-120=60^o$ ($\angle ADC,\angle DAB$ are co-interior angles). $\therefore \angle BCE=\angle ADC=60^o$ (Corresponding angles).
$BC=AD=BE$, $\therefore \Delta BCE$ is isosceles and
$\angle BEC=\angle BCE=60^o$. $\therefore \angle CBE=60^o$. Hence ΔBCE is equilateral.

Q5c At point P, $\dfrac{dy}{dx}=2e^x+3=5$, $\therefore e^x=1$, $x=0$ and $y=2$.
P is $(0,2)$.

Q5di $\Pr(all.red)=\dfrac{{}^{100}C_3}{{}^{300}C_3}=0.0363$

Q5dii $\Pr(at.least.one.not.red)=1-\Pr(all.red)=1-0.0363$
$=0.9637$

Q5diii $\Pr(one.of.each.colour)=\dfrac{\left({}^{100}C_1\right)^3}{{}^{300}C_3}=0.2245$

Q6a

$$\int_0^{20} f(x)dx \approx \frac{20-0}{3\times 4}\left[f(0)+4f(5)+2f(10)+4f(15)+f(20)\right]$$

$$=\frac{5}{3}\left[15+100+44+72+10\right]=\frac{1205}{3}\approx 402$$

Q6bi At $t=10$ min, $V=3600\left(1-\frac{1}{6}\right)^2=2500$ litres

Q6bii At $t=20$ min, $\frac{dV}{dt}=2\times\frac{-1}{60}\times 3600\left(1-\frac{t}{60}\right)$

$$=-120\left(1-\frac{t}{60}\right)=-120\left(1-\frac{20}{60}\right)=-80$$

Drainage rate = 80 litres per min.

Q6biii $\frac{dV}{dt}=2t-120$, the fastest rate is -120 that occurs at $t=0$.

Q6ci Solve $y=x^2$ and $y=12-2x^2$ simultaneously:

$x^2=12-2x^2$, $3x^2=12$, $x=-2$ and $y=4$, or $x=2$ and $y=4$. The intersecting points are $(-2,4)$ and $(2,4)$.

Q6cii $Volume=\int_0^4 \pi(x_1)^2\,dy+\int_4^{12}\pi(x_2)^2\,dy$

$$=\int_0^4 \pi y\,dy+\int \pi\frac{1}{2}(12-y)dy=\left[\frac{\pi y^2}{2}\right]_0^4+\left[\frac{\pi(12-y)^2}{-4}\right]_4^{12}$$

$=8\pi+16\pi=24\pi$ cubic units

Q7ai $a=50000$, $d=2500$, $t_{13}=50000+12(2500)=80000$

Q7aii $a=50000$, $r=1.04$, $t_{13}=50000(1.04)^{12}=80051.61$

Q7aiii Kay: $S_{20}=\frac{50000(1.04^{20}-1)}{0.04}=1488903.93$

Anne: $S_{20}=\frac{20}{2}(2(50000)+19(2500))=1475000$

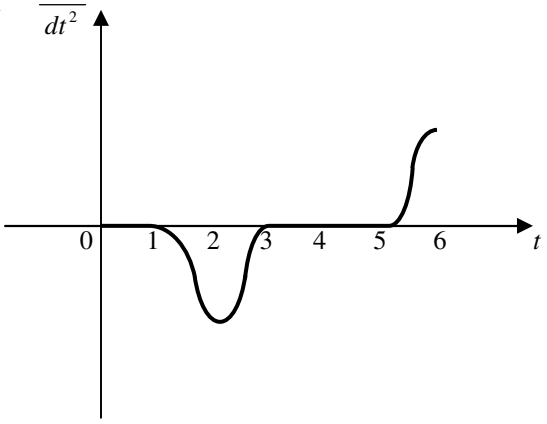
Kay's total exceeds Anne's total by

$1488903.93-1475000=\$13903.93$

Q7bi $t=2$

Q7bii $t=4$

Q7biii $\frac{d^2x}{dt^2}$



Q8ai $x^2=R^2-h^2$, $V=\pi x^2(2h)=2\pi h(R^2-h^2)$

Q8aii Max. volume when

$\frac{dV}{dh}=(R^2-h^2)(2\pi)+(-2h)(2\pi h)=0$,

$2\pi(R^2-3h^2)=0$, $\therefore h=\frac{R}{\sqrt{3}}$.

Q8b Area of shaded region = area of quarter circle –

$$\int_0^1 (x^2-3x+2)dx=\pi-\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_0^1=\pi-\frac{5}{6}$$

Q8ci $A_1=(3\times 10^6)(1.12)-4.8\times 10^5$,

$A_2=A_1(1.12)-4.8\times 10^5=((3\times 10^6)(1.12)-4.8\times 10^5)(1.12)-4.8\times 10^5$

$=(3\times 10^6)(1.12)^2-(4.8\times 10^5)(1+1.12)$

Q8cii $A_n=(3\times 10^6)(1.12)^n-(0.48\times 10^6)(1+1.12+...+1.12^{n-1})$

$$=10^6\left(3(1.12)^n-0.48\left(\frac{1(1.12^n-1)}{0.12}\right)\right)$$

$=10^6(3(1.12)^n-4(1.12)^n+4)=10^6\left[4-(1.12)^n\right]$

Q8ciii After the final payment, $A_n=0$, $10^6\left[4-(1.12)^n\right]=0$,

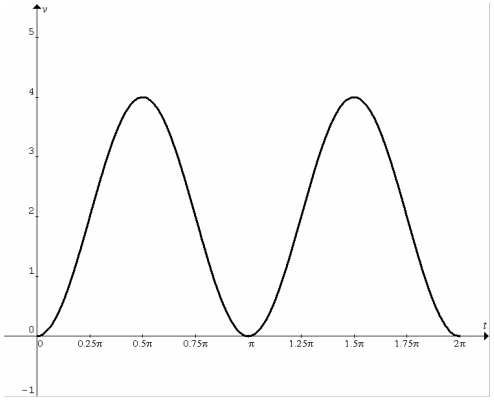
$\therefore (1.12)^n=4$, $n=\frac{\log_e 4}{\log_e 1.12}=12.23$, $\therefore$ year 2017.

Q9ai At $t=0$, $x=0$, $\dot{x}=0$.

At time t, $\dot{x}=\int 4\sin 2t\,dt=-2\cos 2t+C$, $\therefore 0=-2+C$,

i.e. $C=2$. $\therefore \dot{x}=2-2\cos 2t$

Q9aii



First comes to rest at $t=\pi$.

Q9aiii Distance = magnitude of displacement = area under v-t graph from $t=0$ to $t=\pi$.

$$\int_0^{\pi}(2-2\cos 2t)dt=\left[2t-\sin 2t\right]_0^{\pi}=2\pi$$

Q9bi $BD=6\sin\theta$, $\angle DBE=\angle EDF=\angle FEG$ etc.

$DE=BD\sin\theta=6\sin^2\theta$, $EF=DE\sin\theta=6\sin^3\theta$.

Q9bii $BD+EF+GH+...=6\sin\theta+6\sin^3\theta+6\sin^5\theta+...$

$$=6(\sin\theta+\sin^3\theta+\sin^5\theta+...)=6\left(\frac{\sin\theta}{1-\sin^2\theta}\right)=6\left(\frac{\sin\theta}{\cos^2\theta}\right)$$

$$=6\left(\frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}\right)=6\sec\theta\tan\theta$$

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Q10ai $m = \dfrac{\alpha^2 - \beta^2}{\alpha - \beta} = \dfrac{(\alpha - \beta)(\alpha + \beta)}{(\alpha - \beta)} = \alpha + \beta$

$A(\alpha, \alpha^2)$ lies on the line $y = mx + b$, $\therefore \alpha^2 = (\alpha + \beta)\alpha + b$,

$\therefore \alpha\beta = -b$

Q10aii

$AB = \sqrt{(\alpha - \beta)^2 + (\alpha^2 - \beta^2)^2} = \sqrt{(\alpha - \beta)^2\left[1 + (\alpha + \beta)^2\right]}$

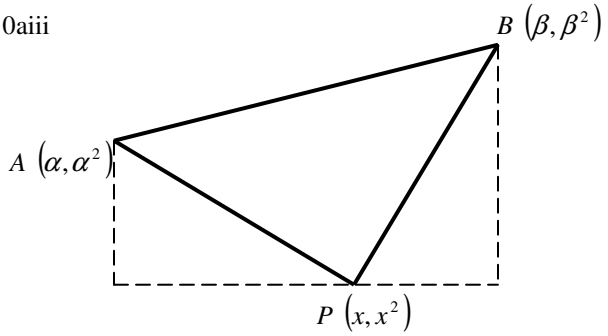
Since $\alpha + \beta = m$, $\therefore m^2 = (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\therefore m^2 = \beta^2 - 2\alpha\beta + \alpha^2 + 4\alpha\beta = (\beta - \alpha)^2 - 4b$

$\therefore (\alpha - \beta)^2 = (\beta - \alpha)^2 = m^2 + 4b$

Hence $AB = \sqrt{(m^2 + 4b)(1 + m^2)}$

Q10aiii



Area of ΔABP = Area of trapezium − areas of two smaller Δs

$= \frac{1}{2}\left[(\alpha^2 - x^2) + (\beta^2 - x^2)\right](\beta - \alpha) - \frac{1}{2}(\alpha^2 - x^2)(x - \alpha) - \frac{1}{2}(\beta^2 - x^2)(\beta - x)$

$= \dfrac{1}{2}\left((\alpha^2 - x^2)(\beta - x) + (\beta^2 - x^2)(x - \alpha)\right)$ expand and simplify

$= \dfrac{1}{2}\left((\beta^2 - \alpha^2)x - (\beta - \alpha)x^2 - (\beta - \alpha)\alpha\beta\right)$

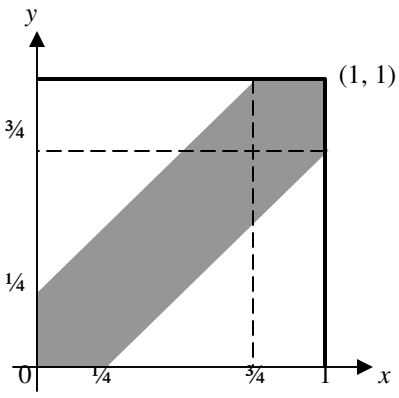
$= \dfrac{1}{2}\left((\beta + \alpha)x - x^2 - \alpha\beta\right)(\beta - \alpha)$

$= \dfrac{1}{2}\left(mx - x^2 + b\right)\sqrt{m^2 + 4b}$

Q10aiv $\dfrac{dA}{dx} = \dfrac{1}{2}\sqrt{m^2 + 4b}\,(m - 2x) = 0$, $\therefore x = \dfrac{m}{2}$ and

$y = \dfrac{m^2}{4}$. Hence P is $\left(\dfrac{m}{2}, \dfrac{m^2}{4}\right)$.

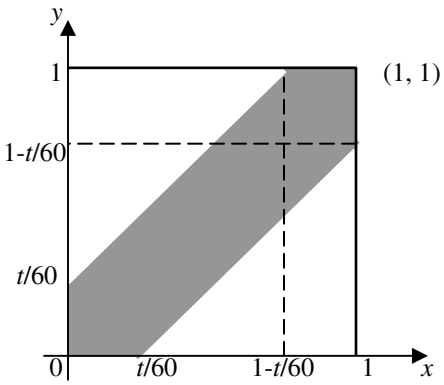
Q10bi



The shaded region satisfies all the inequalities and is inside the square. $\therefore$ they will meet.

Q10bii $\Pr(meet) = \text{area of shaded region} = 1 - \left(\dfrac{3}{4}\right)^2 = \dfrac{7}{16}$

Q10biii



$\Pr(meet) = 1 - \left(1 - \dfrac{t}{60}\right)^2 = 0.5$, $\left(1 - \dfrac{t}{60}\right)^2 = 0.5$,

$1 - \dfrac{t}{60} = 0.7071$, $t = 17.6$ minutes.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.

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