## QUESTION 1

(a)

$$\frac{(3.24)^2}{5.73-2.84} = \frac{10.4976}{2.89} = 3.63 \text{ to 2 dec. pl}$$

(b)

$$\begin{aligned} & 4x - 3(x+5) \\ &= 4x - 3x - 15 \\ &= x - 15 \end{aligned}$$

(c)

$$\begin{aligned} x &= \sqrt{\frac{3V}{h}} \\ &= \sqrt{\frac{3 \times 750}{8.45}} \\ &\approx \sqrt{266.2722} \\ &= 16.32 \text{ to 2 dec. pl.} \end{aligned}$$

(d)

$$3x^2 - 6x + 1 = 0$$

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3 \times 1}}{2 \times 3} \\ &= \frac{6 \pm \sqrt{36-12}}{6} \\ &= \frac{6 \pm \sqrt{24}}{6} \\ &= \frac{6 \pm 2\sqrt{6}}{6} \\ &= \frac{3 \pm \sqrt{6}}{3} \\ &\approx \frac{3 \pm 2.4495}{3} \\ &= \frac{5.4495}{3} \text{ or } \frac{0.5505}{3} \\ &= 1.82 \text{ or } 0.18 \text{ correct to 2 dec. pl.} \end{aligned}$$

(e)

$$\begin{aligned} \frac{2x}{x-5} &= \frac{3}{5} \\ \therefore \quad 5 \times 2x &= 3(x-5) \\ 10x &= 3x - 15 \\ 10x - 3x &= -15 \\ 7x &= -15 \\ x &= -2\tfrac{1}{7} \end{aligned}$$

## QUESTION 2

(a) (i)

$$\frac{d}{dx}(3x^4 - 2x + \frac{1}{x^2} + 1) = 12x^3 - 2 - \frac{2}{x^3}$$

(ii)

$$\frac{d}{dx}(xe^{2x}) = 1.e^{2x} + x.2e^{2x} = (1+2x)e^{2x}$$

(iii)

$$\begin{aligned} \frac{d}{dx}\left(\frac{x}{1+x}\right) &= \frac{(1+x).1 - x.1}{(1+x)^2} \\ &= \frac{1+x-x}{(1+x)^2} \\ &= \frac{1}{(1+x)^2} \end{aligned}$$

(b) (i)

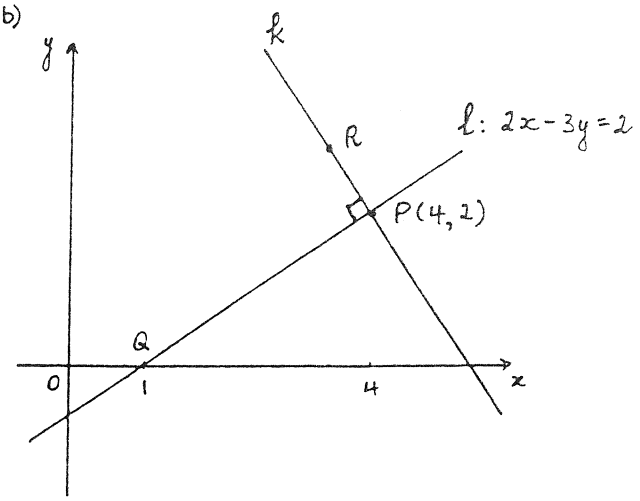
$$\begin{aligned} \int_0^1 (3x^6 + 1)\,dx &= \left[\tfrac{3}{7}x^7 + x\right]_0^1 \\ &= (\tfrac{3}{7} + 1) - (0 + 0) \\ &= 1\tfrac{3}{7} \\ &= 1.4 \text{ to 1 dec. pl.} \end{aligned}$$

(ii)

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \cos 3x\,dx &= \left[\frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{3}\left[\sin\frac{3\pi}{4} - \sin 0\right] \\ &= \frac{1}{3} \times \frac{1}{\sqrt{2}} \\ &= \frac{1}{3\sqrt{2}} \\ &= 0.2 \text{ to 1 dec. pl.} \end{aligned}$$

## QUESTION 3

(a) & (b)



(c) $l: 2x - 3y = 2$ has a gradient of $\frac{2}{3}$.

$\therefore k \perp l$ has gradient of $-\frac{3}{2}$.

$k$ passes through $P(4, 2)$.

$$\begin{aligned} k: \quad y - 2 &= -\frac{3}{2}(x - 4) \\ 2(y - 2) &= -3(x - 4) \\ 2y - 4 &= -3x + 12 \\ 3x + 2y &= 16 \end{aligned}$$

There are two possible positions of $R$, one on each side of $P$. Only one is marked.

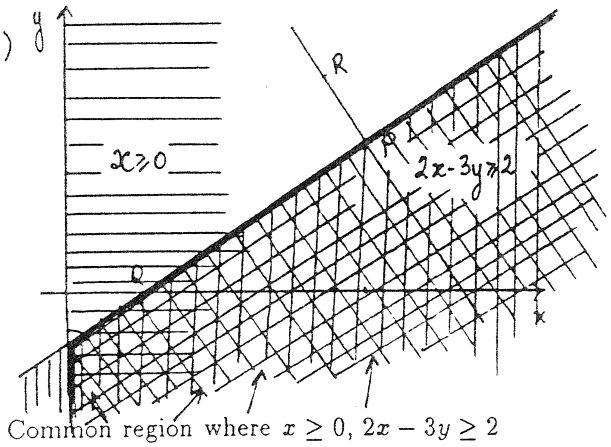
(e)

$$\begin{aligned} PQ &= \sqrt{(4-1)^2 + (2-0)^2} \\ &= \sqrt{3^2 + 2^2} \\ &= \sqrt{13} \end{aligned}$$

(f)

$$\begin{aligned} \text{Area } \triangle PQR &= \tfrac{1}{2} \times QR \times PR \\ &= \tfrac{1}{2} \times \sqrt{13} \times 1 \\ &= \frac{\sqrt{13}}{2} \text{ units}^2 \end{aligned}$$

(g)




Common region where $x \geq 0$, $2x - 3y \geq 2$

## QUESTION 4

(a) (i) $12, 17, 22$ is an A.P. with $a = 12$, $d = 5$.

$u_n = a + (n-1)d$

$u_{25} = 12 + 24 \times 5 = 132$

(ii)

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ &= \frac{25}{2}[12 + 132] \\ &= 1800 \end{aligned}$$

(b)

$$\begin{aligned} x &= t^3 - 9t \\ \dot{x} &= 3t^2 - 9 \\ \ddot{x} &= 6t \end{aligned}$$

(i) Velocity, $\dot{x} = 3t^2 - 9$ metres/sec.

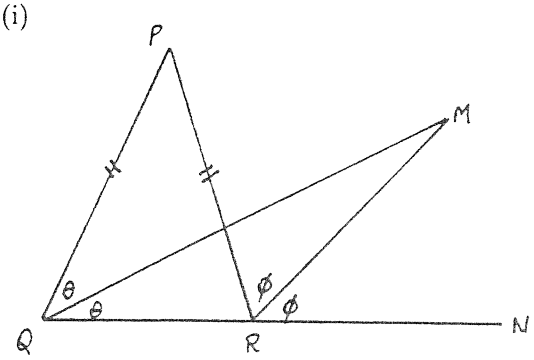
(ii) When $t = 2$, $\dot{x} = 3 \times 2^2 - 9 = 3$ metres/sec

(iii) When $\dot{x} = 0$, $3t^2 - 9 = 0$ $\therefore t = \pm\sqrt{3}$

i.e. after $\sqrt{3} \approx 1.73$ seconds

($t < 0$ has no meaning)

(c) (i)




(ii)

$$\angle QPR = 104°$$

$\therefore \angle PQR + \angle PRQ = 76°$ (angle sum of $\triangle$)

$\therefore \angle PQR = 38°$ ($\triangle PQR$ is isosceles)

(iii) $\angle MRN = \angle MQR + \angle QMR$

(exterior angle of a triangle equals the sum of the interior opposite angles.)

where $\angle MRN = \frac{1}{2}(180° - \angle PRQ)$

(since $MR$ bisects $\angle PRN$)

$= \frac{1}{2}(180° - 38°)$

$= 71°$

where $\angle MQR = \frac{1}{2}\angle PQR = 19°$

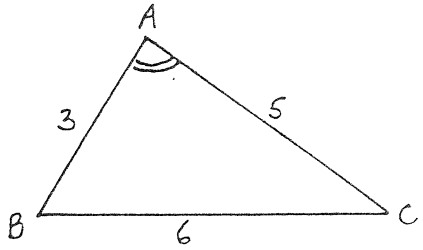
(since $MQ$ bisects $\angle PQR$)

$\therefore 71° = 19° + \angle QMR$

$\therefore \angle QMR = 52°$

## QUESTION 5

(a) (i)




$$\begin{aligned} \cos \angle BAC &= \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5} \\ &= \frac{9 + 25 - 36}{30} \\ &= -\frac{1}{15} \end{aligned}$$

$\therefore \angle BAC = 94°$ (to nearest degree)

(ii)

Area of $\triangle BAC$
$= \frac{1}{2} \times 3 \times 5 \sin \angle A$
$= 7.5 \sin 94°$
$= 7.5\,\text{cm}^2$ (to 2 sig. figs.)

(b)

$$y = x^2 + \frac{5}{x} - 2$$
$$y' = 2x - \frac{5}{x^2}$$

At $P(1,4)$, $y' = 2 \times 1 - \frac{5}{1^2} = -3.$

Tangent: $y - y_1 = m(x - x_1)$
$y - 4 = -3(x - 1)$
$y - 4 = -3x + 3$
$3x + y = 7$

(c)

$$\frac{dy}{dx} = 3x^2 - 2x + 1$$
$$y = x^3 - x^2 + x + c$$

The curve passes through $Q(2,3)$
$\therefore 3 = 8 - 4 + 2 + c$
$3 = 6 + c$
$c = -3$
$\therefore y = x^3 - x^2 + x - 3$

## QUESTION 6

(a) (i)

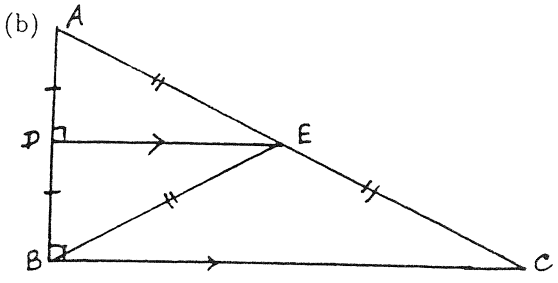
Area shaded region PQRS
= Area sector OQR - area sector OPS
$= \frac{1}{2} \times 25^2 \times \frac{\pi}{3} - \frac{1}{2} \times 15^2 \times \frac{\pi}{3}$
$= \frac{\pi}{6}(25^2 - 15^2)$
$= \frac{200\pi}{3}\,\text{cm}^2$

(N.B. $180° \equiv \pi^c$, $60° \equiv \frac{\pi^c}{3}$)

(ii)

Perimeter of the shaded region PQRS
$= PQ + RS + \text{arc } PS + \text{arc } QR$
$= 10 + 10 + 15 \times \frac{\pi}{3} + 25 \times \frac{\pi}{3}$
$= 20 + \frac{40\pi}{3}\,\text{cm}$

(b)




(i) $\angle ADE = \angle ABC$
(corresponding angles, $DE \parallel BC$)
$\therefore \angle ADE$ is a right angle.

(ii) In $\triangle ADE$ and $\triangle BDE$:

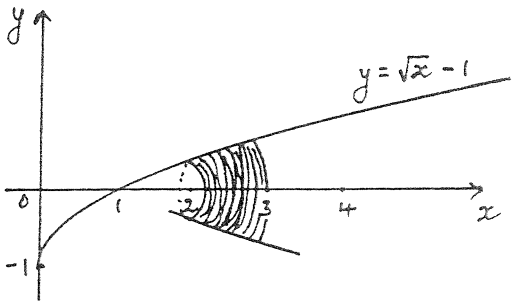
$AD = BD$ (data)
$\angle ADE = \angle BDE$ (right angles)
$DE$ is common
$\therefore \triangle ADE \equiv \triangle BDE$ (SAS test)

(iii) From (ii) $AE = BE$ $(\triangle ADE \equiv \triangle BDE)$
Also $AE = EC$ (parallel lines, $DE$ and $BC$, mark off equal intercepts on transversals since $AD = DB$)

$\therefore BE = EC.$

## QUESTION 7

(a)




$$V = \pi \int_2^3 y^2\,dx$$
$$= \pi \int_2^3 (\sqrt{x} - 1)^2\,dx$$
$$= \pi \int_2^3 x - 2\sqrt{x} + 1\,dx$$
$$= \pi[\tfrac{1}{2}x^2 - \tfrac{4}{3}x^{\frac{3}{2}} + x]_2^3$$
$$= \pi[4\tfrac{1}{2} - \tfrac{4}{3} \times 3\sqrt{3} + 3]$$
$$- \pi[2 - \tfrac{4}{3} \times 2\sqrt{2} + 2]$$
$$= \pi[7\tfrac{1}{2} - 4\sqrt{3} - 4 + \tfrac{8}{3}\sqrt{2}]$$
$$= (3\tfrac{1}{2} - 4\sqrt{3} + \tfrac{8}{3}\sqrt{2})\pi \text{ units}^3$$

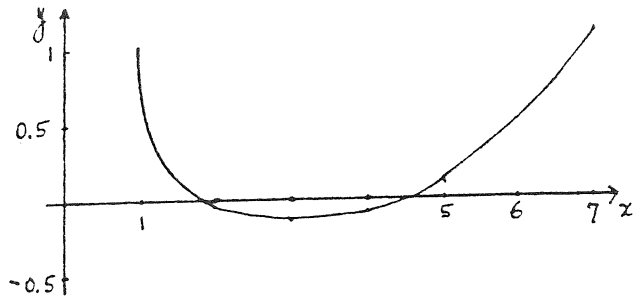
(b) (i)

$$f(x) = x - 3\ln x, \qquad 1 \le x \le 7$$
$$f'(x) = 1 - 3 \times \frac{1}{x}$$
$$= 1 - \frac{3}{x}$$
$$f''(x) = +\frac{3}{x^2}$$

For a turning point,
$f'(x) = 0$
$\therefore \frac{3}{x} = 1$
$x = 3$
$f''(3) = \frac{3}{3^2} > 0$
$f(3) = 3 - 3\ln 3$

$\therefore$ Minimum at $(3, 3 - 3\ln 3) \approx (3, -0.30)$

(ii)




| x | 1 | 2 | 4 | 7 | 5 |
|---|---|---|---|---|---|
| f(x) | 1 | -0.08 | -0.16 | 1.16 | 0.17 |

(iii) $f''(x) > 0$ for $1 \leq x \leq 7$

$\therefore$ Maximum value is either $f(1)$ or $f(7)$

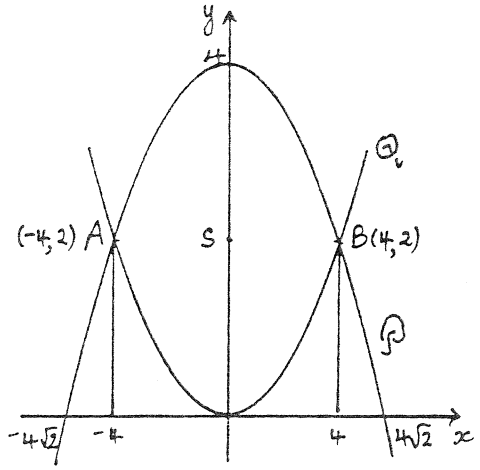
$$f(1) = 1$$
$$f(7) = 1.16$$

$\therefore$ Max. value $= 7 - 3\log 7 \approx 1.16$

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QUESTION 8

(a) (i)




P: $x^2 = 8(4 - y)$

is in the form

$$X^2 = -4a(Y - k)$$

and so has focal length 2.

($\alpha$) $\therefore$ Focus is the point $(0, 2)$

($\beta$) and directrix is the line $y = 6$

($\gamma$) At the $y$–axis, $x = 0 \therefore (0, 4)$

At the $x$–axis, $y = 0 \therefore (4\sqrt{2}, 0), (-4\sqrt{2}, 0)$

(ii)

$x^2 = 8y$ meets $x^2 = 8(4 - y)$

when $8y = 8(4 - y)$

i.e. $y = 4 - y$

i.e. $y = 2$

When $y = 2$, $x^2 = 16$, i.e. $x = \pm 4$

$\therefore A(-4, 2)$, $B(4, 2)$

(iii)

$$\text{Area} = 2\int_0^4 4 - \frac{x^2}{8}\,dx - 2\int_0^4 \frac{x^2}{8}\,dx$$
$$= 2\left[\int_0^4 4 - \frac{x^2}{8} - \frac{x^2}{8}\,dx\right]$$
$$= 2\left[\int_0^4 4 - \frac{x^2}{4}\,dx\right]$$
$$= 2\left[4x - \frac{x^3}{12}\right]_0^4$$
$$= 2\left[\left(16 - \frac{64}{12}\right) - 0\right]$$
$$= 2\left(16 - 5\tfrac{1}{3}\right)$$
$$= 2 \times 10\tfrac{2}{3}$$
$$= 21\tfrac{1}{3} \text{ units}^2$$

(b) $x^4 = 4(x^2 + 8)$

Let $t = x^2$

$\therefore t^2 - 4t - 32 = 0$

$(t - 8)(t + 4) = 0$

$t = 8, -4$

i.e. $x^2 = 8 \Rightarrow x = \pm\sqrt{8}$

or $x^2 = -4 \Rightarrow$ no solutions

$\therefore$ Only real solutions are $x = \pm 2\sqrt{2}$.

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QUESTION 9

(a) (i) There are 3 fences perpendicular to the brick wall each of length $2x$ metres. Total $= 6x$ metres.

There are 2 fences parallel to the brick wall, one of length $7y$ metres, the other of length $y$ metres. Total $= 8y$ metres.

Total fencing $= 56$ metres

$$\therefore 6x + 8y = 56$$
$$8y = 56 - 6x$$
$$\therefore y = \frac{56 - 6x}{8}$$
$$= 7 - \tfrac{6}{8}x$$
$$= 7 - \tfrac{3}{4}x$$

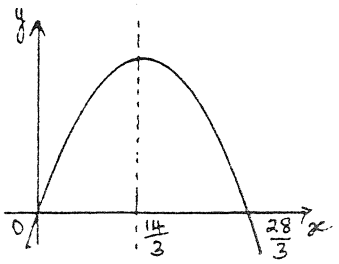
(ii)

$$\text{Area} = 2x \times 7y$$
$$= 2x \times 7(7 - \tfrac{3}{4}x)$$
$$A = 14x(7 - \tfrac{3}{4}x)$$

(iii)

$$A = 14x(7 - \tfrac{3}{4}x)$$
$$= \frac{3}{4} \times 14x\left(\frac{28}{3} - x\right)$$
$$= \frac{21}{2}x\left(\frac{28}{3} - x\right)$$

which is a parabola, with roots of 0, $\frac{28}{3}$ and a max. value at the axis of symmetry, i.e. at $x = \frac{14}{3}$.




When $x = \frac{14}{3}$;

$$\begin{aligned}\text{length} = 7y &= 7(7 - \tfrac{3}{4}x) \\ &= 7(7 - \tfrac{3}{4} \times \tfrac{14}{3}) \\ &= 7(7 - 3\tfrac{1}{2}) \\ &= 7 \times 3\tfrac{1}{2} \\ &= 24\tfrac{1}{2} \text{ metres}\end{aligned}$$

which is half of the available wire.

*Alternately:*

$$\begin{aligned}A &= 14x(7 - \tfrac{3}{4}x) \\ &= 98x - \frac{21}{2}x^2 \\ \frac{dA}{dx} &= 98 - 21x \\ \frac{d^2A}{dx^2} &= -21 < 0\end{aligned}$$

$\therefore$ When $\frac{dA}{dx} = 0$ we have a max. value for A.

(For $0 < x < \frac{28}{3}$, $A'' < 0$ so this value produces a maximum in the relevant domain.)

When $\frac{dA}{dx} = 0$ $\quad 21x = 98$

$$x = \frac{98}{21} = \frac{14}{3}$$

(then as above)

(b) (i)

| 1st die | 2nd die |
|---|---|
| 1 | 1 |
| | 2 |
| | 3 |
| | 4 |
| | 5 |
| | 6 |

etc.

To have a sum larger than 8:

(6,3) (5,4) (4,5) (3,6)
(6,4) (5,5) (4,6)
(6,5) (5,6)
(6,6)

i.e. 10 ways

$$\text{p(sum} > 8) = \frac{10}{36} = \frac{5}{18}$$

(ii) 4 occurs 4 times in these 10 throws. 4 occurs 11 times in the 36 throws:

$\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(1,4),$
$(2,4),(3,4),(5,4),(6,4)\}$

$\therefore$ p(sum > 8, knowing that at least one 4 appears)$= \frac{4}{11}$

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## QUESTION 10

(a) (i) Growth rate = 0.03 = 3% for town B.

(ii) Instantaneous growth rate for town B

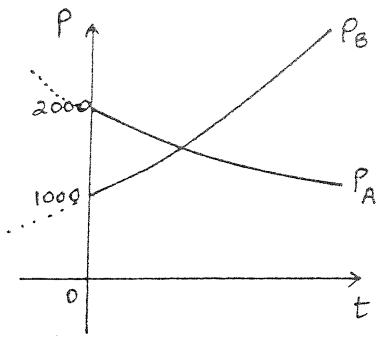
$$= \frac{dP_B}{dt} = 1000 \times 0.03e^{0.03t} = 30e^{0.03t}$$

In 1995, $t = 15$

$$\therefore \frac{dP}{dt} = 30e^{0.45} \approx 47$$

i.e. growth will be (approximately) 47 people in 1995.

(iii)




Population of town B will be greater than population of town A when:

$$1000e^{0.03t} > 2000e^{-0.02t}$$

$$\text{i.e. } e^{0.05t} > 2$$

$$\text{i.e. } 0.05t > \log_e 2$$

$$t > \frac{\log_e 2}{0.05} \approx 13.86$$

i.e. after thirteen years and before the fourteenth year.

i.e. during 1993.

(b) For any quadratic to have real roots $\Delta \geq 0$

$$\text{i.e. } [2l(k+m)]^2 - 4(k^2+l^2)(l^2+m^2) \geq 0$$

$$4l^2(k^2+2km+m^2) - 4(k^2l^2+l^4+k^2m^2+l^2m^2) \geq 0$$

$$\div 4 : k^2l^2 + 2kl^2m + l^2m^2 - k^2l^2 - l^4 - k^2m^2 - l^2m^2 \geq 0$$

$$2kl^2m - l^4 - k^2m^2 \geq 0$$

$$\text{i.e. } l^4 - 2kl^2m + k^2m^2 \leq 0$$

$$(l^2 - km)^2 \leq 0$$

which is never true unless $l^2 = km$.

$\therefore$ The quadratic does not have real roots less $l^2 = km$, in which case the roots are real equal.