HIGHER SCHOOL CERTIFICATE
1990 EXAMINATION PAPER
2/3 UNIT MATHEMATICS

QUESTION ONE

(a) Calculate $\dfrac{171{\cdot}8}{14{\cdot}3\times 5{\cdot}7}$ correct to two decimal places.

(b) The speed of light is 299 725 kilometres per second. Write this number correct to four significant figures.

(c) Factorize $2x^2-7x-15$.

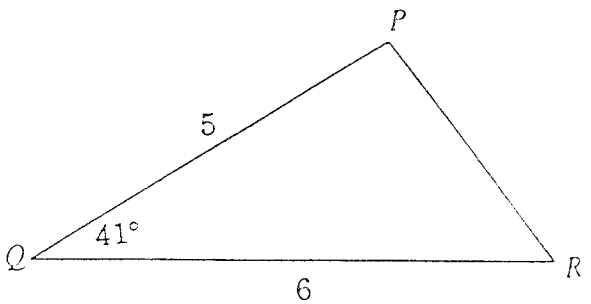
(d) The volume V of a sphere is given by $V=\frac{4}{3}\pi r^3$. If a sphere has volume $5\,\text{cm}^3$, find the radius correct to two decimal places.

(e) Solve $3-(4-x)=5x$.

(f) Mark on a number line the values of x for which $|x-2|<3$.

QUESTION TWO

(a) (figure not drawn to scale)



Find the length of the side PR using the cosine rule and give your answer correct to two decimal places.

(b) (i) Plot on a number plane the point $A(4,3)$. Draw the interval AO where O is the origin.

(ii) Plot the points $B(0,-3)$ and $C(4,0)$ on your diagram.

(iii) Show that the line BC has equation

$$4y=3x-12$$

(iv) Show that $OACB$ is a parallelogram. Give reasons.

(v) Find the area of the parallelogram $OACB$.

(vi) Calculate the length of the diagonal AB.

QUESTION THREE

(a) (i) Rationalize the denominator of

$$\frac{2}{2-\sqrt{3}}.$$

(ii) Find integers a and b such that

$$\frac{2}{2-\sqrt{3}}=a+\sqrt{b}$$

(b) Differentiate:

(i) $7x^5+3$

(ii) $2\sin x+\sqrt{x}$

(iii) $x\ln x$

(c) Find:

(i) $\displaystyle\int\left(\cos x+5x^2\right)dx$

(ii) $\int_0^1 \left(1+e^{-x}\right)dx$

QUESTION FOUR

(a) The sum of the first n terms of a certain arithmetic series is given by

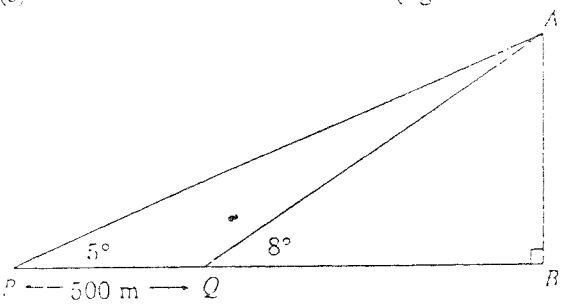
$$S_n = \frac{n(3n+1)}{2}.$$

(i) Calculate S_1 and S_2.

(ii) Find the first three terms of this series.

(iii) Find an expression for the n-th term.

(b) (figure not to scale)



Kaye observes a cliff AB from her yacht at position P. She then sails 500 metres closer to the cliff to position Q. The angle of elevation of the cliff-top from P is $\angle APB = 5°$ and from Q is $\angle AQB = 8°$.

(i) Copy the diagram into your examination booklet and find $\angle PAQ$.

(ii) Use the sine rule to calculate AQ to the nearest metre.

(iii) Hence, or otherwise, find QB to the nearest 10 metres.

QUESTION FIVE

(a) Consider the curve given by $y = 1 + 3x - x^3$, for $-2 \le x \le 3$.

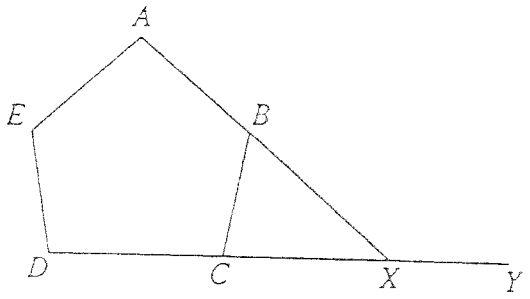
(i) Find the stationary points and determine their nature.

(ii) Find the point of inflexion.

(iii) Sketch the curve for $-2 \le x \le 3$.

(iv) What is the minimum value of y for $-2 \le x \le 3$?

(b) (figure not to scale)



In the diagram, $ABCDE$ is a regular pentagon and AB and DC produced meet at X. The point Y lies on DCX produced.

(i) Copy the diagram into your examination booklet and find the size of $\angle ABC$.

(ii) Find the size of $\angle BXY$ giving reasons.

QUESTION SIX

(a) Find all values of x with $0° \le x \le 360°$ for which

$$\tan x = \frac{1}{\sqrt{3}}.$$

(b) A particle P is at the origin at time $t = 0$ and moves so that its velocity for $t \ge 0$ is given by

$$v = \frac{1}{t+1}.$$

(i) What is the acceleration of P when $t = 1$?

(ii) What is the displacement x of P from the origin when $t = 1$?

(c) Bob tosses two dice with faces numbered 1 to 6. He records the maximum of the two uppermost faces as a score.

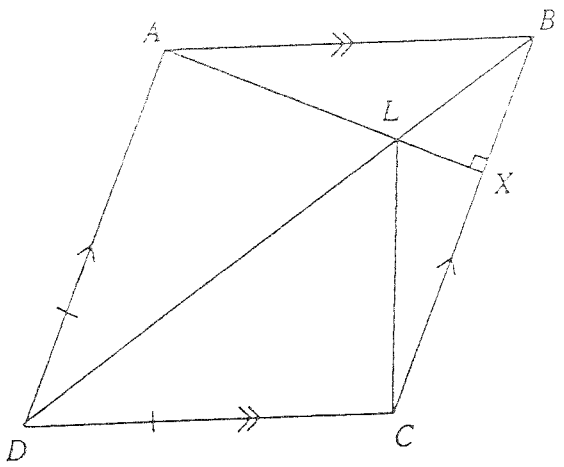
(i) Find the probabiliy that he records the score 1 in a single throw of the two dice.

(ii) Find the probability that he records the scores 1, 1, 1 in three tosses of the two dice.

(iii) Find the probability that he records the score 6 in a single throw of the two dice.

QUESTION SEVEN

(a) (figure not to scale)



$ABCD$ is a rhombus, AX is perpendicular to BC and intersects BD at L.

(i) Copy the diagram into your examination booklet and state why $\angle ADB = \angle CDB$.

(ii) Prove that the triangles ALD and CLD are congruent.

(iii) Show that $\angle DAL$ is a right angle.

(iv) Hence or otherwise find the size of $\angle LCD$.

(b) A box contains 8 red and 11 green marbles. Sue-Mei randomly selects three marbles one at a time and without replacement.

What is the probability that she selects

green, red, green

in that order?

(c) A quantity Q of radium at time t in years is given by $Q = Q_0 e^{-kt}$ where k is a constant and Q_0 is the initial amount of radium at time $t = 0$.

(i) Given that $Q = \frac{1}{2}Q_0$, when $t = 1690$ years, calculate k.

(ii) After how many years does only 20% of the initial amount of radium remain?

QUESTION EIGHT

(a) Find the equation of the normal to the curve $y = x^2 + x$ at the point where $x = 1$.

(b) On a number plane shade in the region given by the two conditions

$$x^2 + y^2 \leq 4$$

and $\quad x + y \geq 1$

(c) It is assumed that the number $N(t)$ of termites in a certain mound at time $t \geq 0$ is given by

$$N(t) = \frac{A}{2 + e^{-t}},$$

where A is a constant, and t is measured in months.

(i) At time $t = 0$, $N(t)$ is estimated at 3×10^5 termites. What is the value of A?

(ii) What is the value of $N(t)$ after one month?

(iii) How many termites would you expect to find in the mound when t is very large?

(iv) Find an expression for the rate at which the number of termites increases at any time t.

QUESTION NINE

(a) Consider the function given by $y = \sin^2 x$.

(i) Copy and complete the following table in your examination booklet.

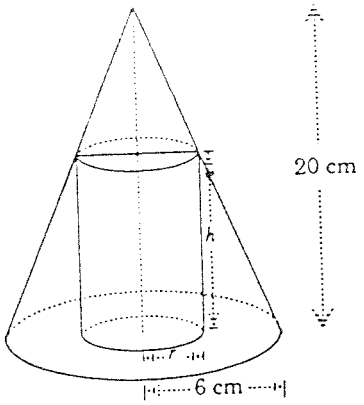
(Note that x is measured in radians.)

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0				

(ii) Apply Simpson's rule with five function values to find an approximation to

$$\int_0^{\pi} \sin^2 x \, dx$$

(b) (figure not to scale)



A cylinder of radium r cm and height h cm is inscribed in a cone with base radius 6 cm and height 20 cm as in the diagram.

(i) Show that the volume V of the cylinder is given by

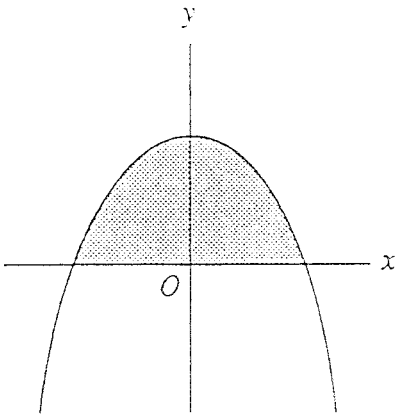
$$V = \frac{10\pi r^2(6-r)}{3}.$$

(ii) Hence find the values of r and h for the cylinder which has maximum volume.

(iii) What is the maximum volume?

QUESTION TEN

(a) (figure not to scale)



The shaded region lying between the curve $y = 1 - x^2$ and the x axis is rotated about the x axis. Find the volume of the solid so formed.

(b) A farmer borrows $80000 to purchase new machinery. The interest is calculated monthly at the rate 2% per month, and is compounded each month. The farmer intends to repay the loan with interest in two equal annual instalments of $M at the end of the first and second years.

(i) How much does the farmer owe at the end of the first month?

(ii) Write an expression involving M for the total amount owed by the farmer after 12 months, just after the first instalment of $M has been paid.

(iii) Find an expression for the amount owed at the end of the second year and deduce that

$$M = \frac{80000 \times (1{\cdot}02)^{24}}{(1.02)^{12} + 1}.$$

(iv) What is the total interest over the two year period?