# HIGHER SCHOOL CERTIFICATE EXAMINATION 1989
## MATHEMATICS - 2/3 UNIT

### QUESTION 1

(a) Find correct to 2 decimal places:

(i) $(2.7)^5$ ;

(ii) $\dfrac{4.6 - 5.9}{4.6 \times 2.3}$ .

(b) Given that $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$, find $S_n$ when $n = 103$, $a = 5$ and $d = 1.2$ .

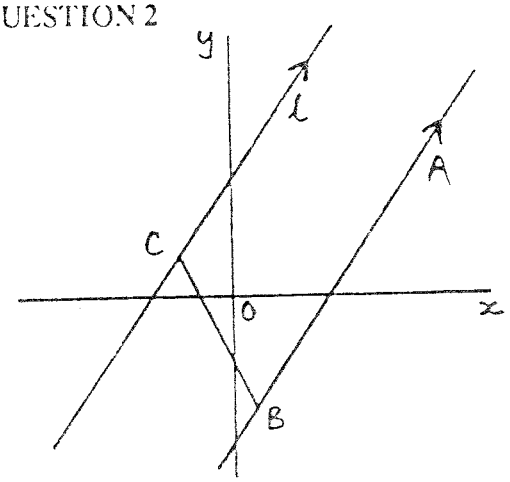
(c) Find the co-ordinates of the midpoint of the interval AB, where A = (2,5) and B = (-4,4).

(d) Solve the equation

$$4(x - 5) = 3 - 2(x - 1) .$$

(e) At a shoe sale, all shoes are to be sold at a discount of 15% off the marked price. What is the cost of a pair of shoes with marked price \$79.95?

### QUESTION 2




The line $l$ passes through C(-1,2) and has equation $y = 2x + 4$. The point B has co-ordinates (1,-6) and the line AB is parallel to $l$.

(i) Copy the diagram into your examination booklet and next to the points B and C write their co-ordinates.

(ii) Find the length of the interval BC.

(iii) Write down the slope of the line $l$ and use your calculator to find the angle $l$ makes with the $x$ axis. Give your answer to the nearest degree.

(iv) Show that AB has equation

$$y = 2x - 8$$

(v) If P is a point which lies on AB and on the line $y = 2$, find the co-ordinates of P.

(vi) Find the length of PC.

(vii) On your diagram, draw the line PC and the perpendicular from B to PC.

(viii) Find the area of the triangle PBC.

### QUESTION 3

(a) Differentiate:

(i) $4x^3 - 7 + \dfrac{5}{x}$

(ii) $x \cos x$.

(b) Find $\displaystyle\int \left(x^2 + \frac{2}{x}\right) dx$

(c) Evaluate $\displaystyle\int_0^1 \left(e^{3x} + 1\right) dx$ .

(d) A sales team sells 1200 calculators in its first month of operation. They plan to increase their sales by 150 calculators each month. How many calculators do they plan to sell:

(i) in the last month of the second year of operation;

(ii) over the entire two year period?

### QUESTION 4

(a)

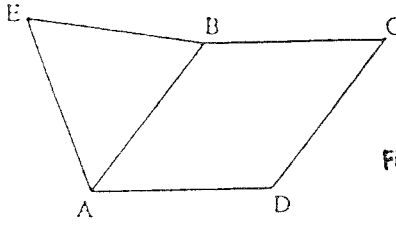



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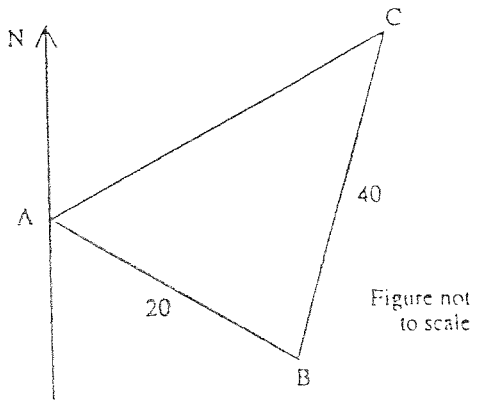
ABCD is a rhombus with $\angle BCD = 48^\circ$
ABE is an equilateral triangle.

(i) In your examination booklet, draw a neat sketch showing this information.

(ii) Find the size of $\angle EAD$, giving reasons for your answer.

(iii) Find the size of $\angle EDA$, giving reasons for your answer.

(b)




Two geologists on a large flat mining claim drive 20km from point A on a bearing of 150°T to point B. They then drive 40km on a bearing of 020°T to point C.

(i) Copy the above diagram into your examination booklet and show that $\angle ABC = 50^\circ$

(ii) Use the cosine rule to find the distance of point C from point A to the nearest kilometre.

(c) In a large school, the student population is 42% male and 58% female. Two students are selected at random to take part in a survey. Find, correct to two decimal places, the probability that :

(i) both are female;

(ii) both are of the same sex;

(iii) they are of different sexes.

## QUESTION 5

(a) A geometric series has second term 6 and the ratio of the seventh term to the sixth term is 3.

(i) Find the common ratio $r$.

(ii) What is the first term $a$ ?

(iii) Calculate the sum of the first 12 terms.

(b) What is the volume of the solid of revolution formed by rotating the curve $y = \sec x$ about the $x$ - axis for $0 \le x \le \frac{\pi}{3}$ ?

(c) A particle moves in a straight line so that its velocity $v$ in metres per second at time $t$ is given by $v = 4 - 2t$.
At time $t = 0$, the particle is at $x = 1$.

(i) Find the displacement $x$ of the particle as a function of $t$.

(ii) When is the particle at rest and what is its acceleration at that time?

(iii) Find the distance the particle travels in the first 4 seconds.

## QUESTION 6

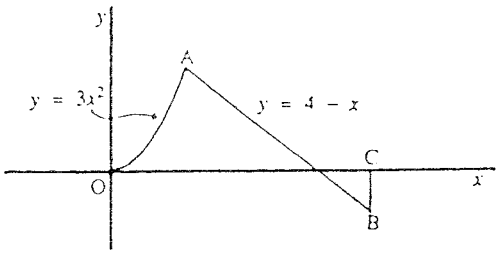
(a) Solve the simultaneous equations

$$4x - y = 3$$
$$10x + 3y = 2$$

(b) (i) Sketch the parabola $\mathcal{P}$, whose focus is the point (2,5) and whose directrix is the line $y = -3$. Indicate on your diagram the vertex and its co-ordinates.

(ii) Find the equation of $\mathcal{P}$.

(c) The shaded region OABC is bounded by the lines $x = 0$, $x = 5$, the curve $y = 3x^2$, the line $y = 4 - x$ and the $x$ axis, as in the diagram.




(i) Show that A has co-ordinates (1,3).

(ii) What is the area of the shaded region OABC?

## QUESTION 7

(a) (i) Find the equation of the tangent to the curve $y = 3 \ln x + 2$ at the point where $x = 1$.

(ii) Find the slope of the normal to the curve at the point where $x = 1$.

(b) The rate of emission E, in tonnes per year, of chloro-fluorocarbons (CFCS) in Australia from 31 October 1989 will be given by

$E = 100 + \left(\frac{40}{1+t}\right)^2$, where $t$ is the time in years

(i) What is the rate of emission E on 31 October 1989?

(ii) What is the rate of emission E on 31 October 1998?

(iii) What value does E approach as the years pass by?

(iv) Draw a sketch of E as a function of $t$.

(v) Calculate the total amount of CFCs emitted in Australia during the years 1989 to 1998.

Question 8.

(a) Solve the equation $2 \cos x = \sqrt{3}$, where $0 \le x \le 2\pi$

(b) Consider the function defined by

$f(x) = e^{2x}(1 - x)$, where $-3 \le x \le 1$

(i) Copy and complete the table of values in your writing booklet. Give values correct to two decimal places.

| $x$ | -3 | -2 | -1 | 0 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 0.01 | 0.05 | | | |

(ii) Differentiate $f(x)$ and hence show that the function has only one stationary point.

(iii) Sketch the curve $y = f(x)$ for $-3 \le x \le 1$

(iv) Without further use of calculus, indicate on your diagram where the curve is concave down.

(v) Using the trapezoidal rule with five function values, approximate the area under the curve $y = f(x)$ for $-3 \le x \le 1$

(vi) From your diagram, decide whether this approximation is an over-estimate or an under-estimate of the true value of the area under the curve. Give a brief reason.

## QUESTION 9

(a) (i) Sketch the function $y = x^2$, for $0 \le x \le 2$ and write down its domain and range.

(ii) If $y$ in (i) is part of an odd function $f(x)$ defined for $-2 \le x \le 2$, sketch on a second diagram the function $f(x)$.

(iii) Write down the value of

$\int_{-2}^{2} f(x)\,dx$, giving a brief reason

for your answer.

(b) Consider the function whose derivative is given by

$$\frac{dy}{dx} = x^2(2x - 1)(x - 1)$$

Determine the nature of the stationary point at $x = 0$.

(c)

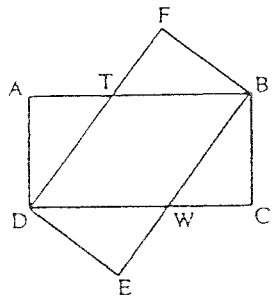



Figure not to scale

ABCD and DEBF are two congruent rectangles with sides 3 and 7 units as in the diagram (AB = DF = 7, AD = DE = 3).

(i) Show that $AT = \frac{20}{7}$

(ii) Find the area of the figure DWBT.

QUESTION 10

(a) In the Jackpot Lottery, the probability of the Jackpot prize being won in any draw is approximately 1 in 50.

(i) What is the probability that the Jackpot prize will be won in each of three consecutive draws?

(ii) How many consecutive draws must be made for it to be 99% certain that a Jackpot prize will have been won?

(b) A cam is formed with cross-section as shown in the figure.

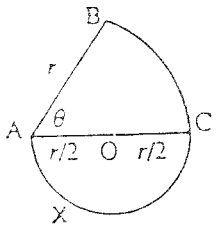



Figure not to scale

The cross-section of a semi-circle AXC centre O and radius $r/2$ and a sector ABC of radius $r$, centre A and angle $\theta$.

(i) What is the perimeter ABCX of the cam in terms of $r$ and $\theta$.?

(ii) If the area of the cross-section of the cam is 1 square unit, show that the perimeter $P$ is given by

$$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right).$$

(iii) Show that the least perimeter occurs when $r^2 = \frac{8}{4+\pi}$

and calculate the value of $\theta$ to the nearest degree.