PREPARED BY

THE NEW SOUTH WALES DEPARTMENT OF EDUCATION

FOR THE

BOARD OF SENIOR SCHOOL STUDIES

1987 HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

2/3 UNIT

Time allowed - Three hours
(Includes reading time)

DIRECTIONS TO CANDIDATES:

- *All* questions may be attempted.
- *All* questions are of equal value.
- All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.
- Standard integrals are printed on page 13 which may be removed for your convenience. Approved slide-rules or silent calculators may be used.
- *Each* question attempted is to be returned in a *separate Writing Booklet* clearly marked Question 1, Question 2 etc. on the cover. Each booklet must show your Candidate's Number and the Centre Number.
- If required, additional Writing Booklets may be obtained from the Examination Supervisor upon request.

Question 1. (*Use a separate book.*)

(i) Find, correct to two decimal places, the value of

$$\frac{5{\cdot}6 \times 4{\cdot}9}{7{\cdot}3 + 4{\cdot}1}.$$

(ii) Solve the equation

$$3(x - 2) = x - 5.$$

(iii) Factorize $3a^2 - 12$.

(iv) Given that $V = \frac{1}{3}\pi R^2 H$ and that $R > 0$, find R if $V = 2000$ and $H = 12$. Give your answer correct to one decimal place.

(v) In 1987 Council rates increased by 7½%. The new rate for a property is \$1735. What was the old rate for this property? Give your answer correct to the nearest dollar.

Question 2. (*Use a separate book.*)

The points P and Q have coordinates (3,-2) and (1,3) respectively.

(a) The line k has equation $4x + 5y - 2 = 0$. Verify that P lies on k.

(b) The line ℓ through Q has gradient $\frac{1}{3}$. Show that the equation of ℓ is $x - 3y + 8 = 0$.

(c) The point of intersection of k and ℓ is R. Find the coordinates of R.

(d) Draw a neat sketch on a number plane showing P, Q, R, k and ℓ.

(e) Find the perpendicular distance of P from ℓ. Leave your answer as a surd.

(f) Find the area of the triangle PQR.

Question 3. (*Use a separate book.*)

(i) Differentiate: (a) $5(4-x)^9$;

(b) $\dfrac{\log_e x}{x}$.

(ii) Find the exact value of:

(a) $\displaystyle\int_0^1 e^{2x}\,dx$;

(b) $\displaystyle\int_1^2 \frac{1}{1+x}\,dx$.

(iii) A parabola has equation $y = x^2 - 4x + 6$.

Find: (a) the equation of its axis of symmetry;

(b) the coordinates of its vertex;

(c) its focal length.

Question 4. (*Use a separate book.*)

(i)

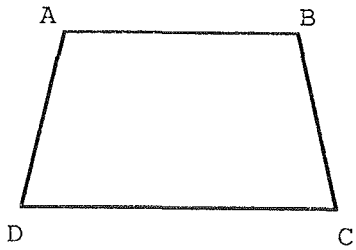


Figure not to scale

ABCD is a quadrilateral, $\angle ADC = \angle BCD$ and $AD = BC$.

(a) In your examination book draw a neat sketch and mark on it all the given information.

(b) Prove that triangle DAC is congruent to triangle CBD.

(c) Why are angles ACD and BDC equal?

(d) Prove that $\angle ADB = \angle ACB$.

(ii)

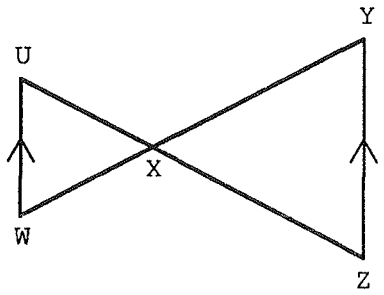


Figure not to scale

The lines UZ and WY intersect at X as in the figure above. $UW \parallel YZ$, UW is 10 cm, WX is 8 cm, UX is 7 cm and XZ = 12 cm.

(a) In your examination book draw a neat sketch and mark on it all the given information.

(b) Prove that the triangles UWX and XYZ are similar.

(c) Hence, or otherwise, find the length of ZY.

Question 5. (*Use a separate book.*)

(i) Find the equation of the tangent to the curve

$$y = 3x + e^{-x}$$

at the point where $x = 0$.

(ii) The function $f(x)$ is given by

$$f(x) = x(x-3)^2.$$

(a) Find the coordinates of the points at which the graph of $y = f(x)$ meets the x-axis.

(b) Find the coordinates of the turning points of $f(x)$ and state whether they are maxima or minima.

(c) Draw a sketch of $y = f(x)$ in the domain $-1 \leqslant x \leqslant 4$.

Question 6. (*Use a separate book.*)

(i) Find the values of x for which

$$|2x - 1| \leqslant 5.$$

(ii) Find the volume of the solid of revolution formed by rotating the curve $y = x + \frac{1}{x}$ about the x-axis between $x = 1$ and $x = 3$.

(iii) (a) Shade on a number plane the region R bounded by the curve $y = 4x - x^2$ and the line $y = 5$, between $x = 0$ and $x = 4$.

(b) Find the area of R.

Question 7. (*Use a separate book.*)

(i) Find the values of m for which the equation

$$4x^2 - mx + 9 = 0$$

has: (a) exactly one real root;

(b) real roots.

(ii) Michele buys five tickets in a raffle in which 20 tickets are sold. Three different tickets are to be drawn out for first, second and third prizes. Using a tree diagram find the probability that:

(a) Michele wins all three prizes;

(b) Michele does not win a prize;

(c) Michele wins at least one prize;

(d) Michele wins exactly one prize.

Question 8. (*Use a separate book.*)

(i)

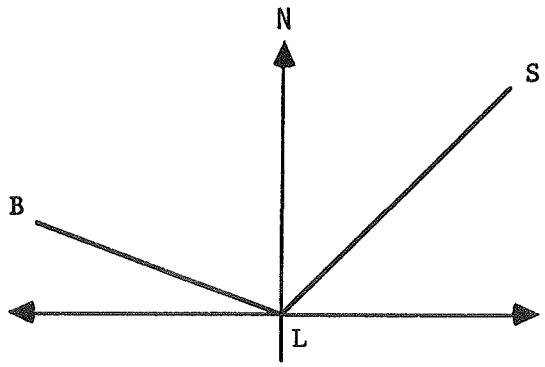


Figure not to scale

From a lighthouse L a ship S bears 053°T and is at a distance of 8 nautical miles. From L a boat B bears 293°T and is at a distance of 6 nautical miles.

(a) Draw a diagram marking on it the information supplied.

(b) Find the distance of ship S from boat B. Give your answer as a surd.

(c) Find the bearing of ship S from boat B. Give your answer to the nearest degree.

(ii) A particle moves in a straight line. At time t seconds its displacement x metres from a fixed point O on the line is given by

$$x = 3 - \cos 2t, \qquad 0 \leqslant t \leqslant 2\pi.$$

(a) Sketch the graph of x as a function of t.

(b) Write down the times when the particle is at rest and the position of the particle at each of these times.

(c) Find the time when the particle first reaches its maximum speed.

Question 9. (*Use a separate book.*)

(i) A person invests \$800 at the beginning of each year in a superannuation fund. Compound interest is paid at 10% per annum on the investment. The first \$800 is to be invested at the beginning of 1988 and the last is to be invested at the beginning of 2017. Calculate to the nearest dollar:

(a) the amount to which the 1988 investment will have grown by the beginning of 2018;

(b) the amount to which the total investment will have grown by the beginning of 2018.

(ii) Tom took a bottle which had 200 millilitres of water in it. He poured more water into it for twenty seconds until it was full. During this time the volume flow rate R of water, in millilitres per second, into the bottle was given by

$$R = 4(20 - t).$$

(a) Find a formula for the volume V of water in the bottle after t seconds where $t \leqslant 20$.

(b) How many millilitres of water were in the bottle when it was full?

(c) How long did it take before the bottle was half full?

Question 10. (*Use a separate book.*)

(i) Find the number which when added to each of 2, 6 and 13 will give a set of three numbers in geometric progression.

(ii) A children's picture book is being designed so that each page contains 320 square centimetres of print and pictures surrounded completely by a white border as illustrated in the figure below.

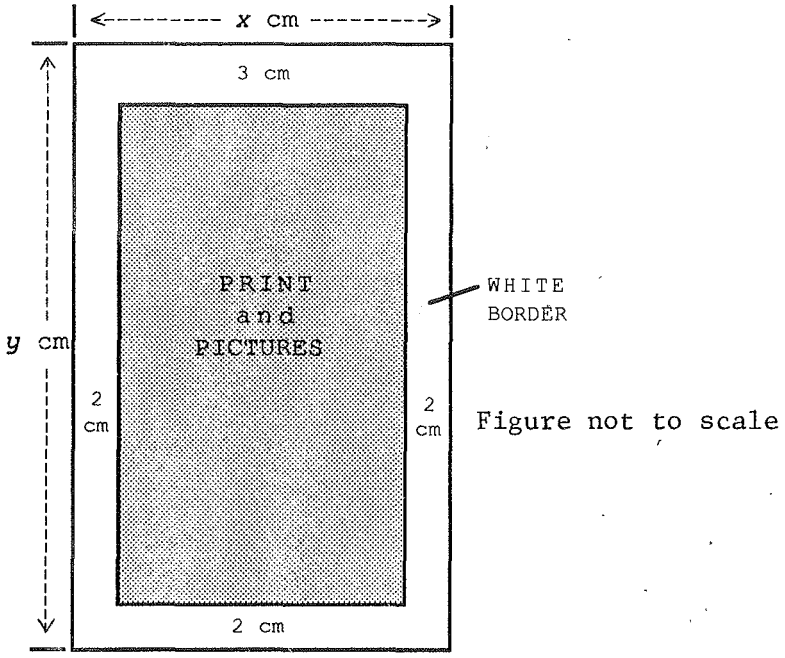


Figure not to scale

Each page is to have a border of width 2 centimetres at the bottom and on each side, as well as a border of width 3 centimetres at the top.

Let the width of a page be x centimetres and its length be y centimetres.

(a) Show that the area A square centimetres of one such page is given by:

$$A = x\left[5 + \frac{320}{(x-4)}\right].$$

(b) Prove that the page which fulfils all the printing requirements given and which has the smallest area is 20 centimetres wide and 25 centimetres long.

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