PREPARED BY

THE NEW SOUTH WALES DEPARTMENT OF EDUCATION

FOR THE

BOARD OF SENIOR SCHOOL STUDIES

HIGHER SCHOOL CERTIFICATE EXAMINATION, 1986

MATHEMATICS

2/3 UNIT

Time allowed—Three hours
(Including reading time)

DIRECTIONS TO CANDIDATES:

- *All* questions may be attempted.
- *All* questions are of equal value.
- All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.
- Standard integrals are printed on page 11 which may be removed for your convenience. Approved slide-rules or silent calculators may be used.
- *Each* question attempted is to be returned in a *separate booklet* clearly marked Question 1, Question 2 etc. on the cover. Each booklet must show your Candidate's Number and the Centre Number.
- If required, additional writing booklets may be obtained from the Examination Supervisor upon request.

Question 1. *(Use a separate book.)* See page 91 for standard integrals.

(i) (a) Find $3^{3.5}$ correct to two decimal places.

(b) The price of an article for sale at \$1.60 is to increase by 20%. What will be the new price of the article?

(c) Given that

$$I = \frac{E}{R + 3r},$$

and $E = 0.6$, $R = 1.2$ and $r = -0.5$, find I.

(ii) In the figure, a vertical pole which is 6.5 metres high casts a horizontal shadow 9.65 metres long. Find the angle of elevation of the sun (correct to the nearest degree).

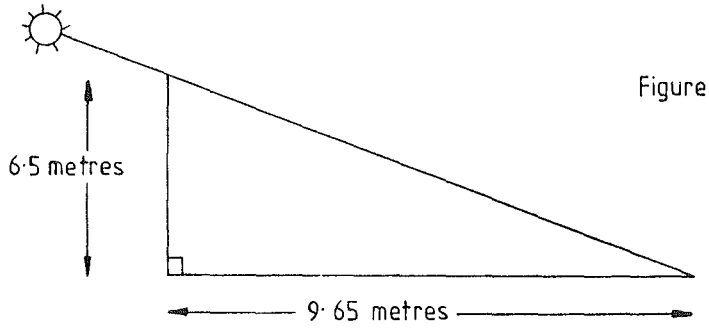


Figure not to scale

(iii) In a certain country town the probability of an adult catching a cold during 1987 is 0.2. Two adults in the town are selected at random. What is the probability that they will both catch colds in 1987?

(iv) Solve the equation

$$3(2x + 1) - 2(3 - x) = 53.$$

Question 2. *(Use a separate book.)*

The coordinates of the points A, B, C are (0,2), (4,0) and (6,-4) respectively.

(a) Find the length AB, and the gradient of AB.

(b) Show that the equation of the line ℓ, drawn through C parallel to AB is

$$x + 2y + 2 = 0.$$

(c) Find the coordinates of D, the point where ℓ intersects the x-axis.

(d) Find the perpendicular distance of the point A from the line ℓ.

(e) Find the area of the quadrilateral ABCD.

Question 3. *(Use a separate book.)*

(i) Differentiate (a) $(x^2 - 1)^{11}$,

(b) $\tan(3x)$.

(ii) Find the equation of the tangent to the curve

$$y = xe^x$$

at the point $(1,e)$.

(iii) Differentiate $$y = \frac{\sin x}{1 + \cos x},$$

and hence show that

$$\frac{dy}{dx} = \frac{1}{1 + \cos x}.$$

Question 4. *(Use a separate book.)*

(i) Evaluate

(a) $\int_1^2 \frac{1}{x}\,dx,$

(b) $\int_0^{\pi/4} \cos(2x)\,dx.$

(ii) The function $p(x)$ is defined by the rule

$$p(x) = (x - 1)(x^2 - 5).$$

(a) Find the real roots of the equation $p(x) = 0$.

(b) Find the coordinates of the turning points of $p(x)$, and state whether they are maxima or minima.

(c) Draw a sketch of the graph $y = p(x)$, in the domain $-3 \leq x \leq 3$.

Question 5. *(Use a separate book.)*

(i) The displacement x metres from the origin at time t seconds, of a particle travelling in a straight line is given by the formula

$$x = t^3 - 21t^2.$$

(a) Find the acceleration of the particle at time t seconds.

(b) Find the times at which the particle is stationary.

(ii)

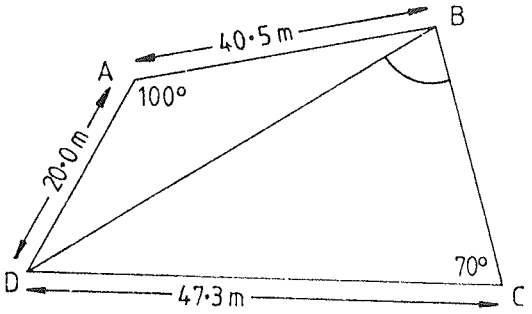


Figure not to scale

lengths in metres

In the above figure, (not drawn to scale), ABCD is a quadrilateral in which AB = 40.5 m, AD = 20.0 m, DC = 47.3 m, $\angle A = 100^\circ$ and $\angle C = 70^\circ$.

(a) Use the cosine rule for triangle ABD to find the length BD.

(b) Use the sine rule to find $\angle DBC$, correct to the nearest degree.

Question 6. *(Use a separate book.)*

(i)

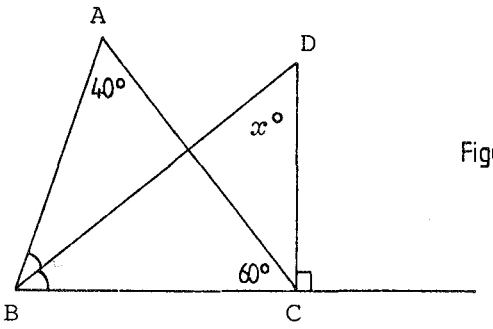


Figure not to scale

In the figure (not drawn to scale), BD bisects $\angle ABC$, DC is perpendicular to BC, $\angle BAC = 40^\circ$, $\angle ACB = 60^\circ$, $\angle BDC = x^\circ$.

(a) Draw a neat sketch of the above diagram in your answer book.

(b) Calculate x, giving reasons for each step in your calculation.

(ii) *(In answering this part, use a new page.)*

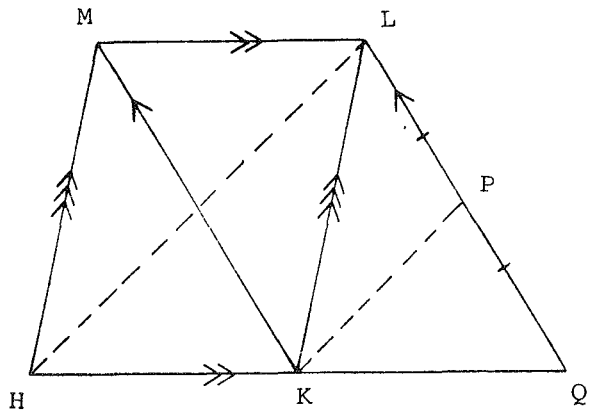


Figure not to scale

In the figure, HKLM is a parallelogram. The line through L, parallel to MK, meets HK produced at Q.

(a) Draw a neat sketch of the diagram in your answer book.

(b) Prove that HK = KQ.

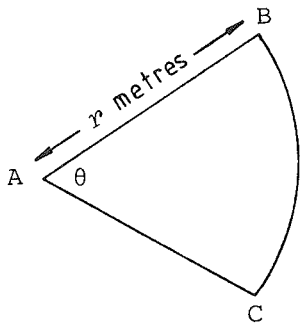
(c) Given that P is the mid-point of LQ, prove that $\angle PKQ = \angle LHQ$.

Question 9. *(Use a separate book.)*

(i) Find all real numbers x which satisfy the equation

$$4x^4 = 4x^2 + 3.$$

(ii)



In the figure, AB and AC are radii of length r metres, of a circle with centre A. The arc BC of the circle subtends an angle θ radians at A.

(a) Write down formulae for

(α) the length of the arc BC,

(β) the area of the sector ABC.

(b) The perimeter of the above figure ABC (i.e. the length AB + arc BC + CA) is 12 metres. Show that the area Y square metres of the sector ABC is given by

$$Y = \frac{72\theta}{(\theta+2)^2}.$$

Hence show that the maximum area of the sector is 9 square metres.

Question 7. *(Use a separate book.)*

(i) (a) Sketch the graph of $y = \cos 2\theta$, for $0^\circ \leq \theta \leq 180^\circ$.

(b) Find all values of θ, for $0^\circ \leq \theta \leq 180^\circ$, such that

$$\cos 2\theta = \frac{1}{2}.$$

(ii) The function $f(x)$, defined for x in the domain $0 \leq x \leq 2$, is given by the rule

$$f(x) = (4 - x^2)^{\frac{1}{4}}; \qquad 0 \leq x \leq 2.$$

(a) Draw up a table of values of $f(x)$ (correct to two decimal places) for

$$x = 0, \tfrac{1}{2}, 1, 1\tfrac{1}{2}, 2.$$

(b) Use this table to draw a sketch of the graph of $y = f(x)$, for $0 \leq x \leq 2$.
[Do not attempt to find the turning points.]

(c) Use the trapezoidal rule with five function values (four equal sub-intervals) to estimate the area between the curve, the x-axis and the y-axis.

Question 8. *(Use a separate book.)*

(i) The first and last terms of an arithmetic series are 10 and 60, respectively, and the sum of the series is 3535. Find:

(a) the number of terms in the series;

(b) the common difference.

(ii) Two identical perfect cubes (similar to dice) each having faces numbered 0, 1, 2, 3, 4, 5 are rolled. A score for the roll is determined as the <u>product</u> of the two numbers on the two uppermost faces.

(a) The cubes are rolled once. What is the probability that the score is

(α) 0 ?

(β) at least 16 ?

(b) If the cubes are rolled twice and the scores for each roll are added, what is the probability of a combined score of at least 41?

Question 10. *(Use a separate book.)*

(i) The rate of increase of a population $P(t)$ of persons in a certain country is governed by the equation

$$\frac{dP}{dt} = kP$$

where k is a constant, and t is the time in years. The population of the country doubles every twenty years.

(a) Find k.

(b) In which year will the country reach a population three times that it had at the beginning of 1960?

(c) Given that at the beginning of 1960 the population was 15.1 million, what will be the population at the beginning of the year 2010?

(ii) (a) Find the points of intersection of the curve $y = 4 - \sqrt{2x}$ with the x-and y-axes.

(b) The area enclosed by the curve $y=4 - \sqrt{2x}$, the x-axis, and the y-axis, is rotated about the y-axis. Find the volume of the solid of rotation so formed.

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