PREPARED BY

THE NEW SOUTH WALES DEPARTMENT OF EDUCATION

FOR THE

BOARD OF SENIOR SCHOOL STUDIES

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**HIGHER SCHOOL CERTIFICATE EXAMINATION, 1984**

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# MATHEMATICS

**2 UNIT/3 UNIT COMMON PAPER**

*Time allowed—Three hours*

*All* questions may be attempted

*All* questions are of equal value

In every question, all necessary working should be shown

Marks may not be awarded for careless or badly arranged work

**Standard integrals are printed on page 9 which may be removed for your convenience; approved slide-rules or silent calculators may be used**

The answers to the ten questions in this paper are to be returned in separate writing booklets clearly marked Question 1, Question 2, etc., on the cover. Each booklet must show your Candidate's Number and the Centre Number on the cover.

If required, additional writing booklets may be obtained from the examination supervisor upon request

**Question 1.** (*Use a separate book.*) See page 91 for standard integrals.

(i) Express 357.896

(a) in scientific notation,

(b) rounded off correct to the nearest whole number,

(c) rounded off correct to the nearest one-hundredth.

(ii) Find the exact value of

(a) $-2 - (-3)$,

(b) $2.9181 \times 10^{23}$ divided by the product of $2.13 \times 10^{11}$ and $1.37 \times 10^{9}$,

(c) $\frac{1}{5}(t^3 - t^2 + 1)$ when $t = -3$,

(d) $36^{\frac{1}{4}} \times 36^{\frac{1}{4}}$.

(iii) Solve the equations

(a) $T^3 = 4913$,

(b) $2w = -7(1000 - w)$.

**Question 2.** (*Use a separate book.*)

(i) Solve the equation

$$\frac{3x - 1}{5x + 1} = \frac{3x - 2}{5x + 2}.$$

(ii) Find the derivative of each of

(a) $(x + 3)(x^2 - 4)$,

(b) $\sin 2x + 3 \tan x$,

(c) $\log_e (1 + e^x)$.

(iii) Find

(a) $\int_0^{\pi/4} (1 + \tan^2 \theta)\, d\theta$,

(b) the value of $k$ such that $\int_1^k \frac{4}{x^2}\, dx = 3$.

**Question 3.** (*Use a separate book.*)

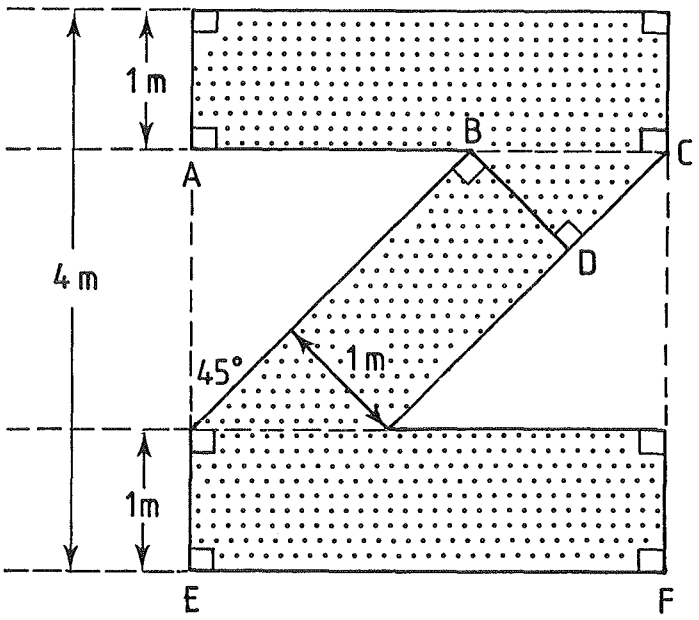
(i) A car engine is running at 5000 revolutions per minute.

(a) How many revolutions are made in one hour?

(b) If this engine uses 10 litres of fuel in one hour, how much fuel does it use every revolution?

(c) This engine has four cylinders and fuel is supplied to two of them each revolution. How much fuel (in millilitres rounded off correct to three decimal places) is supplied to a cylinder when it receives fuel during a revolution?

(ii)




The figure above shows the letter $Z$ which is painted on a wall as part of a sign for a Zoo.

(a) How long is the segment $AB$ of the letter?

(b) The right angled isosceles triangle $BCD$ has equal sides of length one metre. How long is its hypotenuse?

(c) Use the results of (a) and (b) to show that the length $EF$ shown on the diagram is $(2 + \sqrt{2})$ metres.

(d) Find, in square metres, the area painted.

(e) Find, in metres, the perimeter of this painted area.

**Question 4.** (*Use a separate book.*)

(i) The lines $L_1$, $L_2$ have equations

$$L_1: x - 2y - 1 = 0,$$

$$L_2: 2x + y + 18 = 0.$$

(a) Find the shortest distance from $P(8,4)$ to $L_1$.

(b) Show that the lines intersect at $Q(-7, -4)$.

(c) Find the distance $PQ$.

(ii) Given that

$$Q(x) = ax^2 + bx + c, \text{ for all } x,$$

and that

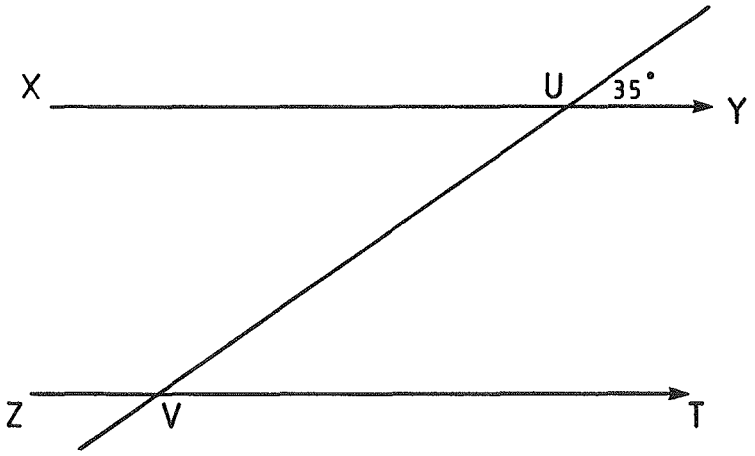
$$Q(0) = 4,\ Q(1) = 23,\ Q(-1) = 1,$$

determine the constants $a$, $b$, $c$.

Show that the equation $Q(x) = 0$ has no (real) solutions.

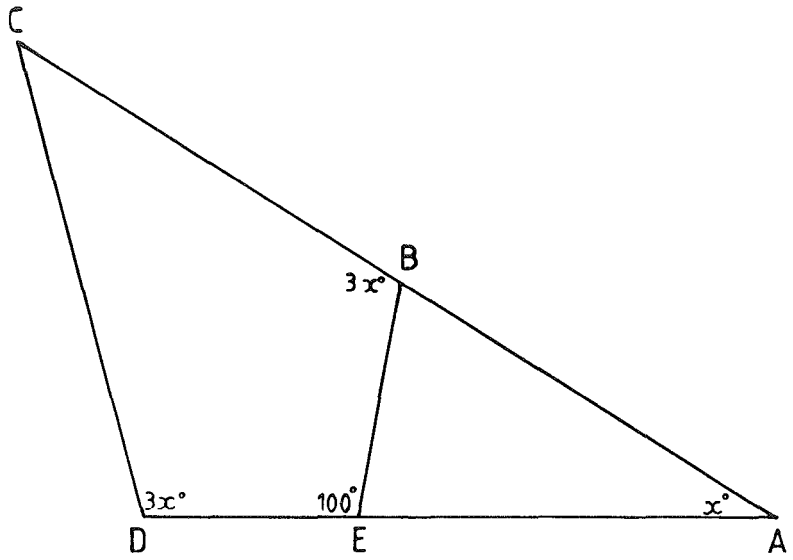
**Question 5.** (*Use a separate book.*)

(i) In the diagram, $XY//ZT$.




Find the size of $UVT$ and give a reason for your answer.

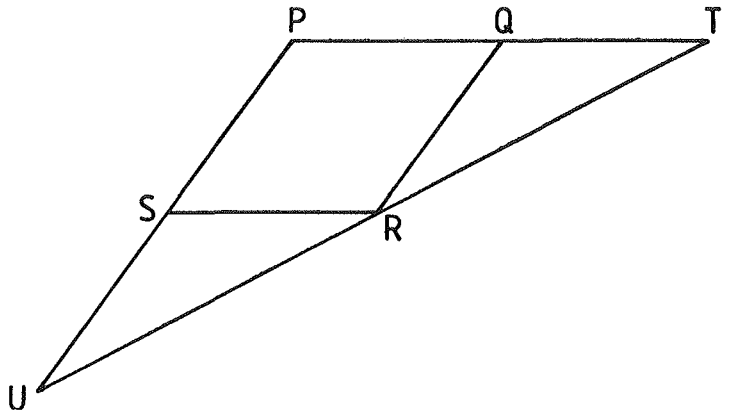
(ii)




As shown in the figure (which is not to scale), $B,E$ lie on the sides $AC$, $DA$ respectively of $\triangle ACD$. Use the information shown on the figure to find the value of $x$ and hence give $A\hat{C}D$ in degrees. Give reasons for your answer.

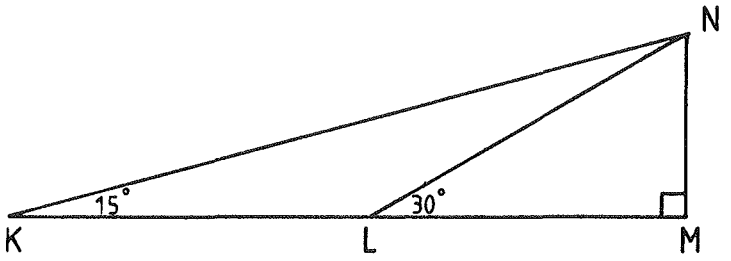
**(Question 5 is continued on next page.)**

(iii)




$PQRS$ is a parallelogram. $PQ$ is produced beyond $Q$ to $T$ so that $QT = QR$ and $PS$ is produced beyond $S$ to $U$ so that $SU = PS$. It is now discovered that $T$, $R$, and $U$ are collinear. Prove $PQRS$ is a rhombus.

(iv)




(a) Find $K\hat{N}L$, $L\hat{N}M$ in degrees and show that $KL = LN$.

(b) Given that the length of $NM$ is 1 unit, write down the lengths of $LM$, $LN$ and deduce that

$$\tan 15° = 2 - \sqrt{3}.$$

**Question 6.** (*Use a separate book.*)

(i) For the domain $-1 \leqslant x \leqslant 3$, determine the maximum value and the minimum value of each of the functions defined by

$$f(x) = 3x^2, \qquad g(x) = 4x - x^2.$$

Find all values of $x$ in the interval $-1 \leqslant x \leqslant 3$ for which $f(x) = g(x)$.

Sketch the curves $y = f(x)$ and $y = g(x)$, for $-1 \leqslant x \leqslant 3$, on the same diagram. Indicate clearly the points where maximum and minimum values are attained and the points of intersection of the two curves.

Calculate the area of the region totally enclosed by portions of the two curves.

(ii) Use the trapezoidal rule with five equal subintervals to estimate

$$\int_3^8 \log_e (x^2)\, dx,$$

rounding your answer to one decimal place.

**Question 7.** (*Use a separate book.*)

(i) From a point $O$ the point $P$ bears 120° from the North and is 12.3 km away. The point $Q$ is 15.2 km South West of $O$. Mark the relative positions of $O$, $P$, $Q$ on a sketch. Calculate the distance $PQ$ in kilometres (rounded off correct to one decimal place).

(ii) On 1st January, 1957, a person joins a superannuation fund by investing \$3000 at 9% p.a. compound interest. A similar amount is invested at the beginning of each subsequent year until the person retires on 31st December, 1984.

(a) Show that the accumulated value of the investment at the date of retirement is \$369406 correct to the nearest dollar.

(b) If this amount is taken as a lump sum payment, which is taxable at the rate of 30 cents in every dollar in excess of \$50000, then how much does the person receive after tax?

**Question 8.** (*Use a separate book.*)

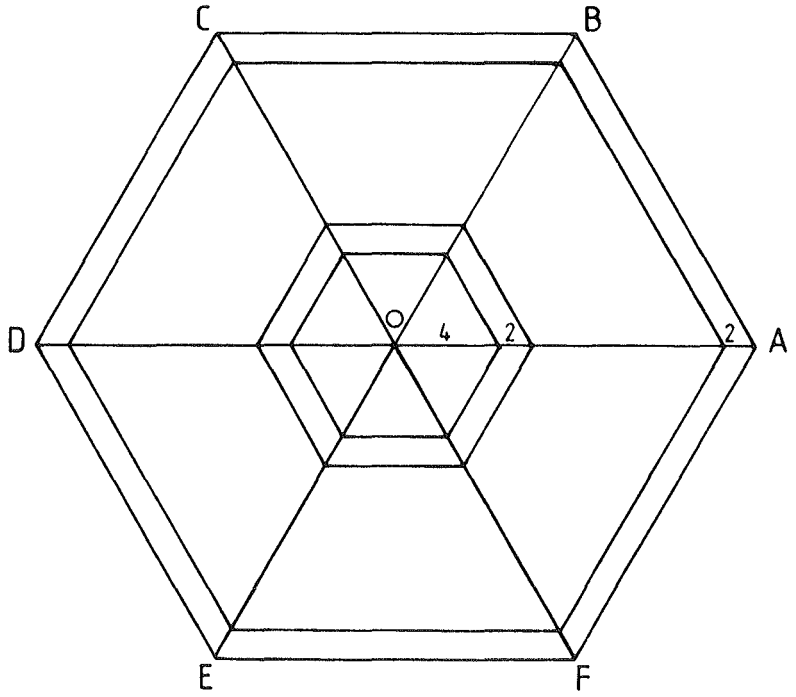
(i) The area under the curve $y = \sqrt{9 - x^2}$, $-3 \leqslant x \leqslant 3$, is rotated about the $x$-axis. Find the volume of the solid of revolution thus obtained. Describe the shape of this solid.

(ii) Write down the formula for:

(a) the $n$th term of an arithmetic series with first term $a$ and common difference $d$,

(b) the sum of the first $n$ terms of this series.

A particular spider's web consists of a series of regular hexagons with a common centre $O$, held together by rays through $O$, as in the figure, where only some of the hexagons are shown.




NOT TO SCALE LENGTHS IN CM.

The vertices of the smallest hexagon are 4 cm from $O$, the vertices of the next are 2 cm further away and they continue at 2 cm intervals along the rays until the vertices of the last hexagon $ABCDEF$ are 60 cm from $O$.

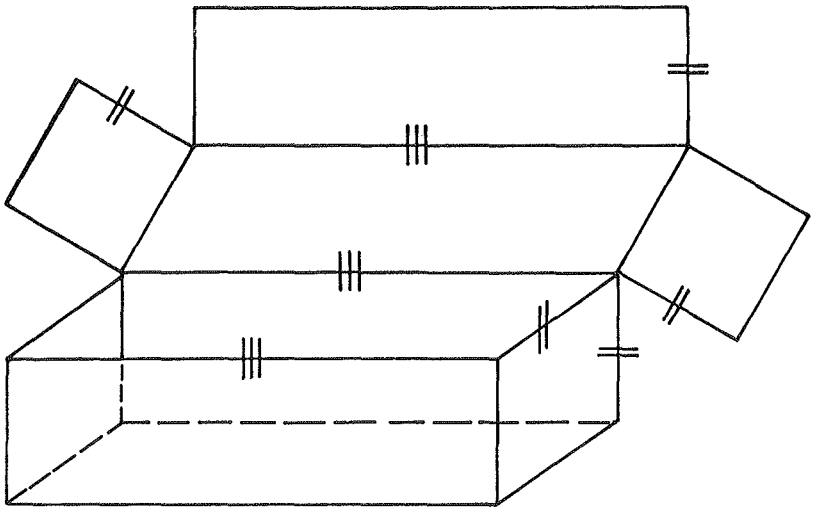
(c) How many hexagons are there?

(d) What is the length, in cm, of the perimeter of the smallest hexagon?

(e) What is the total length of thread used by the spider in making this web (including the six rays from $O$)?

**Question 9.** (*Use a separate book.*)

(i) Farmer Brown packs her unsalted butter in rectangular cardboard boxes with square ends and overlapping lids which exactly cover the open top, the square ends, and the front face. Each box uses 0.27 square metres of cardboard. What is the maximum volume of a box?



(ii) A small town has 1000 telephone numbers consisting of the numbers 5000 to 5999 inclusive.

(a) How many of these numbers end in 3?

(b) You remember only that a friend's telephone number ends in 2 and has 4 as one (or both) of its two middle digits. What is the probability that the number is 5142?

**Question 10.** (*Use a separate book.*)

(i) A particle $P$ moves along a straight line so that at time $t$ ($\geqslant 0$) its displacement from a fixed point $O$ on that line is given by

$$x(t) = 3t^2(4 + t^3)^{-1}.$$

(a) Find the velocity of the particle at time $t$.

(b) Find the times when the particle is momentarily at rest.

(c) Show that $P$ is in (exactly) the same position at both the times $t_1 = 1, t_2 = 2 + 2\sqrt{2}$. Give a brief description of the way the *displacement* changes over the time period from $t_1$ to $t_2$.

(d) Describe the motion of the particle as $t$ increases without bound.

(e) Find the maximum displacement from $O$.

(ii) The line $y = mx$ is tangent to the curve $y = e^{3x}$. Find $m$.

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0.$$

$$\int \frac{1}{x} dx = \log_e x,\ x > 0.$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\ a \neq 0.$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax,\ a \neq 0.$$

$$\int \sin ax\, dx = \frac{-1}{a} \cos ax,\ a \neq 0.$$

$$\int \sec^2 ax\, dx = \frac{1}{a} \tan ax,\ a \neq 0.$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a} \sec ax,\ a \neq 0.$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\ a \neq 0.$$

$$\int \frac{1}{\sqrt{(a^2 - x^2)}} dx = \sin^{-1} \frac{x}{a},\ a > 0,\ -a < x < a.$$

$$\int \frac{1}{\sqrt{(x^2 - a^2)}} dx = \log_e \{x + \sqrt{(x^2 - a^2)}\},\ |x| > |a|.$$

$$\int \frac{1}{\sqrt{(x^2 + a^2)}} dx = \log_e \{x + \sqrt{(x^2 + a^2)}\}.$$

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