NEW SOUTH WALES

Department of Education

HIGHER SCHOOL CERTIFICATE EXAMINATION, 1979

Mathematics

2 UNIT COURSE

Time allowed—Three hours

All questions may be attempted

All questions are of equal value

In every question, all necessary working should be shown

Marks will be deducted for careless or badly arranged work

Mathematical tables will be supplied

Approved slide-rules or calculators may be used

The answers to the ten questions in this paper are to be returned in separate writing booklets clearly marked Question 1, Question 2, etc., on the cover

If required, additional writing booklets may be obtained from the examination supervisor upon request

Question 1. (*Use a separate book.*)

(i) Differentiate with respect to x:

(a) $4x^3 + x\sqrt{x}$;

(b) e^{x^2+1};

(c) $\dfrac{2x - 1}{3x + 2}$.

(ii) Find:

(a) $\displaystyle\int_2^4 \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$;

(b) $\displaystyle\int_0^{\pi} (2 \sin x - \sin 2x)\, dx$.

(iii) If $\displaystyle\int_0^a (4 - 2x)\, dx = 4$, find the value of a.

(iv) If $\dfrac{dy}{dt} = 6t^2 - 2$ and $y = 1$ when $t = 1$, find y when $t = 2$.

Question 2. (*Use a separate book.*)

(i) Write in simplest form the expression

$$2x - (x - 2).$$

(ii) Solve each of the following:

(a) $-3 \leqslant 1 - x < 4$;

(b) $|x + 1| = 4$.

(iii) Rationalise the denominator and express in simple surd form

$$\frac{2\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

(iv) A function is defined by the following rule:

$$f(x) = \begin{cases} 0 & \text{if } x \leqslant -2, \\ -1 & \text{if } -2 < x < 0, \\ x & \text{if } x \geqslant 0. \end{cases}$$

Find:

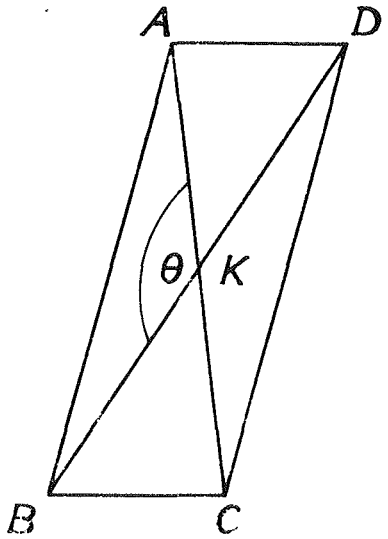
(a) $f(-2) + f(-1) + f(0)$;

(b) $f(a^2)$.

Question 3. (*Use a separate book.*)

(i) Find the exact value of $\frac{2}{15} - \frac{275}{1000}$ as a fraction in its lowest terms.

(ii) Express $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ as a fraction in its lowest terms.



(iii) $ABCD$ is a parallelogram (as shown in the diagram) in which the lengths of AB and AD are 3 cm and 1 cm respectively and the angle ABC is 60°; also the diagonals cut at K and the angle AKB is denoted by θ.

Calculate the exact values (giving answers as rational numbers or surds) of:

(a) the length of the diagonal AC;

(b) the length of the diagonal BD;

(c) $\cos\theta$.

Question 4. (*Use a separate book.*)

The point $Q\ (-2,1)$ lies on the line k whose equation is $9x - 2y + 20 = 0$.

The point $R\ (4,-2)$ lies on the line l whose equation is $3x + y - 10 = 0$.

(i) Show that k and l intersect at a point P on the y-axis.

(ii) Find the equation of the line m which joins Q and R.

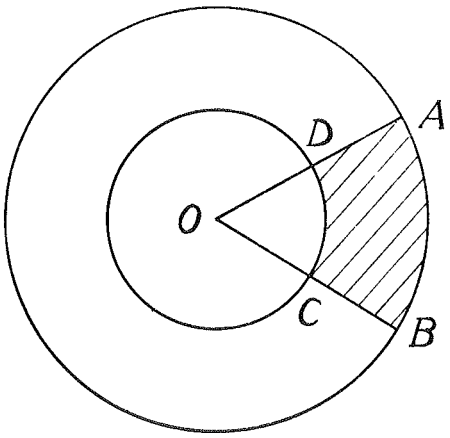
(iii) Show, by shading on a sketch (not on graph paper), the region defined by the three inequalities

$$\begin{aligned} 9x - 2y + 20 &\geqslant 0, \\ 3x + y - 10 &\leqslant 0, \\ x + 2y &\geqslant 0. \end{aligned}$$

(iv) Find, as a surd, the perpendicular distance from P to m.

(v) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines k, l, and m.

Question 5.



(i) The diagram shows two concentric circles centre 0 and radii 20 cm and 10 cm respectively. ODA and OCB are straight lines and the angle between OA and OB is 60°.

Given $\pi = 3.142$, find, correct to 3 significant figures:

(a) the perimeter of the shaded region $ABCD$;

(b) the area of the shaded region $ABCD$.

(ii) (a) Find the focus and directrix of the parabola

$$x^2 = 4y\,.$$

(b) A line is drawn through the focus of the above parabola parallel to the x-axis. The region bounded by this line and the parabola is rotated about the y-axis. Find the volume of the solid of revolution.

(iii) Find the values of m for which the equation

$$x^2 + (m - 2)x + 4 = 0$$

has:

(a) equal roots;

(b) no real roots.

Question 6. (*Use a separate book.*)

(i) Find the minimum value of

$$x + \frac{900}{x}$$

for $x \geqslant 0$, giving reasons for your answer.

(ii) A cargo service operates by running a ship between port A and port B at a constant speed of v kilometres per hour.

For a given v, the *cost per hour of running of the ship* is

$$9000 + 10v^2 \text{ dollars.}$$

Find the value of v which minimizes the cost of the trip.

Question 7. (*Use a separate book.*)

(i) Two ordinary dice, with the numbers 1 to 6 on their faces, are thrown. What is the probability that:

(a) they both show 6?

(b) they show a 1 and a 6?

(c) at least one of them shows a 1?

(d) they show a total of six?

(ii) On a destroyer there are two lines of defence against aircraft attack. These are a surface-to-air missile and a 15 mm rapid-firing gun. The probability of success in hitting an attacking aircraft with each line of defence is respectively 0.9 and 0.8.

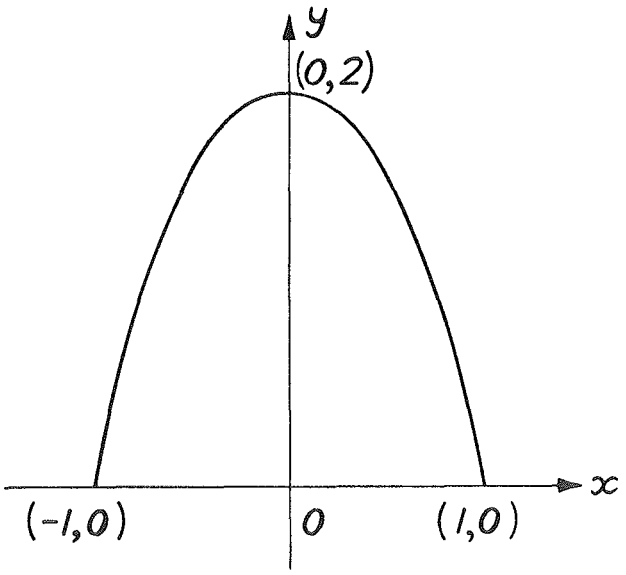
Find the probability of hitting an attacking aircraft before it penetrates both defences.

Question 8. (*Use a separate book.*)

(i) Draw sketches (not on graph paper) of

(a) $y = 3 \cos \frac{1}{2}x$ from $x = -\pi$ to $x = \pi$;

(b) $y = \cos \pi x$ from $x = -1$ to $x = 1$.



(ii) An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of an arc of either a cosine curve or a parabola, as illustrated on axes above.

(a) If the arch is made in the shape of the curve

$$y = 2 \cos \frac{\pi}{2}x,$$

find the area of the window. (Your answer may be left in terms of π.)

(b) If the arch is made the shape of an arc of a parabola, find the equation of the parabola and the area of the window.

Question 9. (*Use a separate book.*)

(i) Given $\log_2 3 = 1.15851$, find, correct to two decimal places:

(a) $\log_2 9$;

(b) $\log_2 12$.

(ii) Show that the tangents to the graphs $y = \log_e x$ and $y = \log_e 2x$ at the points where $x = 1$ are parallel.

(iii) A vertical line $x = a$ is drawn to cross the two graphs $y = \log_e x$ and $y = \log_e 2x$ at points A and B. Find the distance AB and show that it is constant (that is, it does not depend on the value of a).

(iv) A horizontal line is drawn to cross the two graphs $y = e^x$ and $y = \frac{1}{2}e^x$ at points C and D. Show that the distance CD is constant (that is, it does not depend on the position where the horizontal line is drawn).

Question 10. (*Use a separate book.*)

(i) The first three terms of an arithmetic series are 50, 43, 36.

(a) Write down a formula for the nth term.

(b) If the last term of the series is -27, how many terms are there in the series?

(c) Find the sum of the series.

(ii) A loan of \$1000 is to be repaid by equal annual instalments, repayments commencing at the end of the first year of the loan. Interest, at the rate of 10 per cent, is calculated each year on the balance owing at the beginning of that year, and added to that balance.

If the annual instalment is P dollars, prove that:

(a) the amount owing at the beginning of the second year of the loan is $(1100 - P)$ dollars;

(b) the amount owing at the beginning of the third year of the loan is $(1210 - 2.1P)$ dollars;

(c) if the loan (including interest charges) is exactly repaid at the end of n years; then

$$P = \frac{100}{1 - \frac{1}{(1.1)^n}}$$

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