NEW SOUTH WALES

Department of Education

HIGHER SCHOOL CERTIFICATE EXAMINATION, 1977

Mathematics

2 UNIT COURSE

Time allowed—Three hours

All questions may be attempted

All questions are of equal value

In every question, all necessary working should be shown

Marks will be deducted for careless or badly arranged work

Mathematical tables will be supplied

Slide rules may be used

The answers to the ten questions in this paper are to be returned in separate writing booklets clearly marked Question 1, Question 2, etc., on the cover

Question 1. (*Use a separate book.*)

(i) (a) Express $\sqrt{45} + \sqrt{80}$ in the form $a\sqrt{5}$ and find a.

(b) Express $\dfrac{4 + \sqrt{3}}{2 + \sqrt{3}}$ in the form $b - 2\sqrt{3}$ and find b.

(ii) Find the discriminant of the following equation and state whether the roots are real

$$2x^2 + 3x + 5 = 0.$$

(iii) In the Cartesian plane, indicate, by shading, the region whose points simultaneously satisfy the inequalities:

$$x + y \leqslant 3,$$
$$x - y \geqslant 1.$$

Question 2. (*Use a separate book.*)

(i) Find the axis of symmetry and the vertex of the parabola

$$y = 5x^2 + 10x + 2.$$

(ii) Find the acute angle between the lines

$$2x + 3y = 13,$$
$$2x - 5y = 5.$$

(iii) Find the perpendicular distance from the point $(2, -6)$ to the line

$$4x - 3y + 9 = 0.$$

Question 3. (*Use a separate book.*)

(i) Differentiate with respect to x:

(a) $2x^2 - 6x + 7$;

(b) $\sqrt{(x^2 + 5)}$;

(c) $\sin\left(\frac{x}{2}\right)$.

(ii) Evaluate $\lim\limits_{x \to 0} \frac{\sin 3x}{x}$.

(iii) Find the minimum value of $x \log_e x$ (where $x > 0$) correct to three decimal places.

Question 4. (*Use a separate book.*)

(i) If $\log_e 3 \doteqdot 1{\cdot}0986123$ and $\log_e 2 \doteqdot 0{\cdot}6931472$ evaluate $\log_e 12$ correct to 5 decimal places.

(ii) Solve the inequality $|x - 5| > 9$.

(iii) If $f(x) = ax^2 + bx + c$ find the value of $f(x) - f(-x)$.

(iv) The area under the curve $y = 2x - x^2$ between $x = 0$ and $x = 2$ is rotated about the x-axis through one complete revolution. Find the volume of the solid so formed.

Question 5. (*Use a separate book.*)

(i) Find primitives (i.e. indefinite integrals) of:

(a) $x^{\frac{1}{3}}$;

(b) $x^{-\frac{1}{3}}$;

(c) e^{2x}.

(ii) Evaluate:

(a) $\int_2^7 \frac{4}{x + 3}\, dx$;

(b) $\int_0^{\frac{\pi}{3}} \sin 2x\, dx$.

(iii) Use the mid-ordinate rule with one strip to find an approximate value of

$$\int_2^4 \frac{\log_e x}{x}\, dx .$$

Question 6. (*Use a separate book.*)

(i) If $\sin A + \sin 40° = 2 \sin 50° \cos 10°$ and A is acute, find A.

(ii) A triangular piece of land is enclosed by three sections of straight road. Two of these sections are 2.1 km and 3.4 km and meet at an angle of 67°. Find the length of the third side to two significant figures.

(iii) Draw separate sketches (not on graph paper) of:

(a) $y = 2 \cos x$ for $-2\pi \leqslant x \leqslant 2\pi$;

(b) $y = 2 + 2 \cos x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Question 7. (*Use a separate book.*)

(i) Show that the points $A(3,-1)$, $B(7,2)$ and $C(1,10)$ are the vertices of a right angled triangle. Also find the area of the triangle ABC.

(ii) The tangent at the point $L(3,4)$ on the curve $y = \frac{12}{x}$ cuts the x-axis at P and the y-axis at Q. Find the co-ordinates of P and Q. In what ratio does L divide the interval PQ?

Question 8. (*Use a separate book.*)

(i) A particle moves along the x-axis with acceleration at time t equal to $12(t + 3)^2$.

If the particle is initially at rest at the origin,

$$\left(\text{i.e. } v = \frac{dx}{dt} = 0 \text{ at } t = 0\right),$$

find its position when $t = 2$.

(ii) Show that of all rectangles with perimeter 36 cm the one which has maximum area is a square.

Question 9. (*Use a separate book.*)

(i) Comment on the following reasoning:

"*When two coins are tossed they can either fall as two heads or as two tails or as a head and tail. As there are three possibilities the probability of two heads is* $\frac{1}{3}$."

(ii) If three coins are tossed what is the probability of their falling as two heads and a tail?

(iii) In a certain school the senior students are divided into two classes A and B. Class A contains 11 female and 9 male students and class B contains 8 female and 12 male students. A certain occasion requires two speakers, one from each class. These are selected at random. Find the probabilities that:

(a) both speakers are male;

(b) both speakers are female;

(c) one speaker is male and one is female.

Question 10. (*Use a separate book.*)

(i) (a) Sum the arithmetic series $1 + 3 + 5 + \ldots$ to n terms.

(b) Sum the arithmetic series $2 + 4 + 6 + \ldots$ to n terms.

(c) Find the sum of the first 2 000 terms of the series

$$1 - 2 + 3 - 4 + 5 - 6 + \ldots + (-1)^{n+1} n + \ldots .$$

(ii) If $\frac{dy}{dt} = -4y$ and $y = 10$ when $t = 0$

(a) express y as a function of t;

(b) find the value of t for which $y = 0{\cdot}1$;

(c) if $y = \frac{dx}{dt}$ and $x = 10$ when $t = 0$, find an expression for x in terms of t.

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