NEW SOUTH WALES

Department of Education

HIGHER SCHOOL CERTIFICATE EXAMINATION, 1976

Mathematics

2 UNIT COURSE

Time allowed—Three hours

All questions may be attempted

In every question, all necessary working should be shown

Marks will be deducted for careless or badly arranged work

Mathematical tables will be supplied

Slide-rules may be used

The answers to the ten questions in this paper are to be returned in separate writing booklets clearly marked Question 1, Question 2, etc., on the cover

Question 1. (12 marks) (*Use a separate book.*)

(i) Find the value of sin 1·7 (N.B. radians).

(ii) Simplify and evaluate:

(a) cot 65°. sin 65° ;

(b) $\sin^2 20° + \sin^2 70°$.

(iii) Draw separate sketches (showing the main features—NOT on graph paper) of:

(a) $x^2 + y^2 = 36$;

(b) $y = x^2 + 4$;

(c) $y = 2^x$;

(d) $y = 2 \sin x$ for $0 \leqslant x \leqslant 2\pi$.

Question 2. (9 marks) (*Use a separate book.*)

(i) Find the equation of the line passing through the point (2,7) and parallel to the line $2x - 3y = 8$.

(ii) On a sketch indicate, by suitable shading and labelling, the region

$\{(x,y) : y \geqslant x\} \cap \{(x,y) : 0 \leqslant y \leqslant 6\}$.

(iii) The three lines $3x - y = 6$, $2x + y = 14$ and $y = 0$ enclose a triangle. Find the area of this triangle.

Question 3. (9 marks) (*Use a separate book.*)

(i) Solve the following inequalities:

(a) $-3x < 10$;

(b) $\frac{1}{x} > \frac{1}{6}$;

(c) $|x + 1| < 7$.

(ii) Solve the equation $x + \frac{1}{x} = 3$. (Leave answers in surd form.)

(iii) Express $\frac{6 + \sqrt{2}}{3 + \sqrt{2}}$ as a fraction with a rational denominator.

Question 4. (10 marks) (*Use a separate book.*)

(i) Differentiate:

(a) $2x^3 + 7x$;

(b) $x^{\frac{1}{3}}$;

(c) $\sin^2 x$;

(d) $\frac{x}{\cos x}$.

(ii) Find the greatest and least values of y where $y = x^3 - 12x + 20$ and x is restricted to the interval $-3 \leqslant x \leqslant 5$.

Question 5. (10 marks) (*Use a separate book.*)

(i) Find primitives (i.e. indefinite integrals) of:

(a) $12x^2 - 4$;

(b) e^{2x};

(c) $\sin 6x$.

(ii) Evaluate:

(a) $\int_0^{\frac{\pi}{2}} \cos x \, dx$;

(b) $\int_9^{13} \frac{dx}{x - 7}$.

(iii) The following table gives values of $f(x) = x \log x$:

x	1	2	3	4	5
$f(x)$	0	1·39	3·30	5·55	8·05

Use Simpson's rule with these five values to find an approximate value of

$\int_1^5 x \log x \, dx$.

Question 6. (10 marks) (*Use a separate book.*)

(i) Given that k is a positive number specify (a) the smallest and (b) the largest of the following numbers:

$$2^{-\frac{1}{2}k},\ 2^{\frac{1}{2}k},\ 2^{k},\ 2^{-k}.$$

(ii) Find the sum of the first 100 terms of the arithmetic progression whose first two terms are 100 and 95.

(iii) By considering $0\dot{4}\dot{7}$ (i.e. 0·474747. . .) as the sum of an infinite geometric progression, or otherwise, find a simple equivalent fraction for $0\dot{4}\dot{7}$.

Question 7. (10 marks) (*Use a separate book.*)

(i) If $\sin 57° + \sin 23° = 2 \sin 40° \cos A$, write down an exact value of A.

(ii) The sides of a triangle are 7 cm, 5 cm and 4 cm. Find the size of the angle opposite the largest side.

(iii) For the parabola $x^2 = 8y$, find:

(a) the focal length;

(b) the co-ordinates of the focus;

(c) the equation of the directrix;

(d) the equation of the tangent to the parabola at the point $(-4,2)$.

Question 8. (10 marks) (*Use a separate book.*)

(i) A particle is moving in the x-axis. At time t its position x is given by

$$x = t + 1 + \frac{1}{1+t}.$$

Find:

(a) the initial position (i.e. at time $t = 0$) ;

(b) the velocity at time $t = 2$;

(c) the acceleration at time t.

(ii) A particle is moving in the x-axis. It started from rest at time $t = 0$ from the point $x = 7$.

If its acceleration at time t is $2 + 6t$, find the position of the particle when $t = 3$.

Question 9. (10 marks) (*Use a separate book.*)

(i) Comment briefly on the following statement, giving reasons for your view:

"There are twelve teams in a football competition. The probability that a particular team will win that competition is $\frac{1}{12}$."

(ii) One hundred tickets are to be sold in a raffle. Two different tickets are to be drawn out for first and second prizes respectively. A man buys ten tickets. Find the probabilities that:

(a) he wins the first prize;

(b) he wins both prizes;

(c) he wins neither prize;

(d) he wins at least one prize.

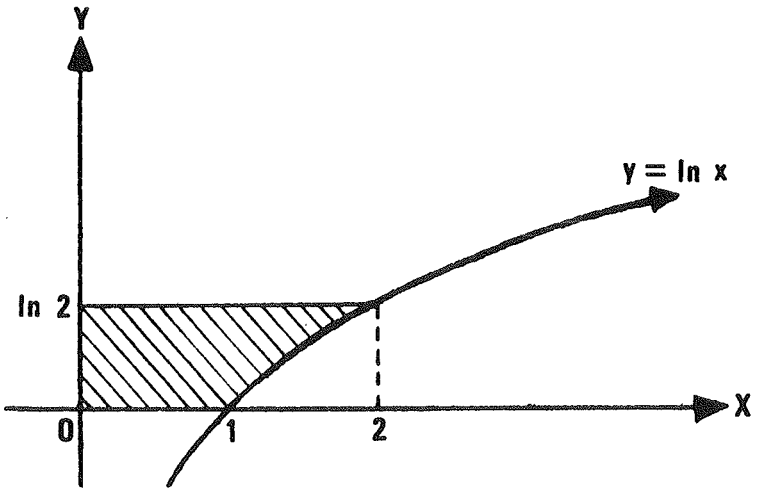
Question 10. (10 marks) (*Use a separate book.*)

(i) On the same diagram (NOT on graph paper) sketch the graphs of

$y = e^{3x}$ and $y = e^{-x}$.

(ii) Find the angle between the curves in part (i) at their point of intersection.
(i.e. the angle between their tangents at this point.)

(iii)



Find the area shaded in the above diagram. (NOTE—ln x denotes $\log_e x$.)

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