N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1974
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Arrange the following numbers in order of increasing magnitude: $\sqrt{2}$; $\frac{7}{5}$; $1\frac{3}{7}$; $1.\dot{4}$.

(ii) A circle, of radius 5 units, has its centre at the point (-3, 4). What are the coordinates of the two points at which the circle cuts the y-axis?

(iii) What is the area of the square on the hypotenuse of a triangle with the other sides 1 and $(1 + \sqrt{2})$ units? (Give the answer in surd form.)

(iv) Comment briefly on the following statement, giving justification for your view:

"As there are 26 letters in the alphabet, if I choose any letter on a page of print at random, the probability that is an 'e' is $\frac{1}{26}$."

QUESTION 2 (9 Marks)

(i) Differentiate (a) $x^3 - 5x + 2$ (b) $(2x - 3)^5$ (c) $x^2 \cos x$

(ii) Write down a primitive function of (a) $x^{1/2} + 1$ (b) x^{-13}

(iii) Evaluate $\int_1^4 (3x - 2)dx$

QUESTION 3 (9 Marks)

(i) The first two terms of a geometric sequence are 9 and 6.

(a) What is the third term?

(b) What is the nth term?

(ii) For the arithmetic sequence 1, 4, 7, ...

(a) find the 50th term

(b) calculate the sum of the first 50 terms.

(iii) A plant is 50 cm high when first observed. In the first week of observation it grows 10 cm, and in each succeeding week the growth in height is 80% of the previous week's growth. If this pattern of growth continues, what will be its ultimate height?

QUESTION 4 (10 Marks)

(i) A particle is moving in a straight line and its velocity at time t is given by $v = 3\sin 2t$.

(a) Calculate (to 3 significant figures) its velocity when $t = 0.1$.

(b) Write down an expression for its acceleration at time t.

(ii) For another particle moving in a straight line its acceleration at time t seconds ($t > 0$) is given by $a = 4 - \frac{8}{t^3}$ m/s^2. If the velocity is 10 m/s when $t = 2$, what is its velocity when $t = 3$?

QUESTION 5 (10 Marks)

(i) For the graph of $y = x^3 + 3x^2 - 9x$ find

(a) the coordinates of the stationary points

(b) the set of values of x for which the graph is concave upwards.

(ii) The parabola $y = ax^2 - c$ and the circle $x^2 + y^2 = 16$ meet on both the x and y axes. If a and c are both positive, what are their values?

QUESTION 6 (10 Marks)

(i) A field is triangular, with two sides of 80 metres and 40 metres enclosing an angle of 130°.

(a) Calculate the area of the field.

(b) Use the cosine rule to calculate the length of the third side to the nearest metre.

(ii) An astronaut observes that the moon, which has a diameter of 3500 km, subtends an angle of 1° at his eye. Calculate the approximate distance of the astronaut from the moon.

QUESTION 7 (10 Marks)

(i) Find and test all real solutions of the equation $|x + 1| = 2x + 7$

(ii) (a) Sketch, using the same axes, but not on graph paper, $y = x^2 - 2x - 3$ and $xy = -1$

(b) Write down an equation whose roots are given by the abscissae of the points of intersection of these two graphs.

QUESTION 8 (10 Marks)

(i) It is known from experience that 90% of the eggs from a certain farm are fresh. If a cook chooses 3 of these eggs, what is the probability:

(a) that all 3 are fresh?

(b) that at least one is stale?

(ii) Find the equation of the locus of the point which moves so that its distance from the x-axis is always equal to its distance from the point (1, 3). Name the locus.

QUESTION 9 (10 Marks)

(i) What is the equation of the tangent to the curve $y = \log_e x$ at the point (1, 0)?

(ii) The area under the curve $y = \frac{1}{x}$ between $x = 1$ and $x = b$ is equal to 1 square unit. What is the value of b?

§ (iii) Use the mid-ordinate rule with one strip to estimate $\int_{3.7}^{4.3} \frac{1}{1 + \sqrt{x}}\,dx$

QUESTION 10 (10 Marks)

(i) Find the volume swept out when the region under the curve $y = e^x$, for $1 \le x \le 1.5$ is rotated about the x-axis, giving your answer to 3 significant figures.

(ii) The number of bacteria (P) in a colony, after t minutes is given by $P = 1000e^{0.007t}$. Find:

(a) the number of bacteria when $t = 10$

(b) the rate at which the colony is increasing when $t = 10$.