N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1971
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Given that θ is in the second quadrant and that $\tan\theta = -\frac{3}{4}$, find $\cos\theta$.

(ii) Draw a diagram showing a region in which both the inequalities $x + y + 1 > 0$, $1 - x - y > 0$ hold simultaneously.

(iii) Simplify $(1 - \cos^2\theta)(1 + \cot^2\theta)$.

(iv) Differentiate (a) e^{-x} (b) $\tan 2x$ (c) $\frac{1}{x^2}$

QUESTION 2 (9 Marks)

(i) Find the second derivative of $e^x \cos x$.

(ii) Find all solutions of $x + 1 = |5 - 3x|$

(iii) Is the quadratic $2 + 4x + x^2$ positive definite? Justify your answer.

QUESTION 3 (9 Marks)

(i) A triangle ABC has sides AB, AC of lengths 4 units, 6 units respectively, and the measure of the angle A is 30°. Write down the area of ΔABC.

(ii) A ball is picked at random from a bag containing 2 red and 3 black balls; then a ball is picked from a second bag containing 4 red, 2 black and 3 green balls. What is the probability that balls drawn are red? (Leave your answer as a fraction.)

(iii) An integer n, $1 \le n \le 100$, is picked at random. What is the probability that n is divisible by 7? (Leave your answer as a fraction.)

QUESTION 4 (10 Marks)

(i) Calculate $\int_0^{0.4} \cos x \, dx$ approximately by Simpson's Rule from the following table:

x	0	0.1	0.2	0.3	0.4
$\cos x$	1	0.995	0.980	0.955	0.921

Given that 0.4 radians equals 22° 55', what is the value (correct to four decimal places) of this integral?

(ii) Write down primitives (indefinite integrals) of:

(a) $\frac{4x}{1+2x^2}$ (b) $\frac{1}{x\sqrt{x}}$

QUESTION 5 (10 Marks)

The first 4 terms of four sequences are given below:

(a) 3, 1, 4, 5 (b) 4, -2, 1, $-\frac{1}{2}$ (c) -3, -1, 1, 3 (d) 3, 5, 7, 11

(i) In each case state whether the sequence could be a geometric progression, an arithmetic progression, or could not be either.

(ii) For the sequence (c) assume that it is in fact of the type you consider possible and write down the tenth term and the sum of the first ten terms.

QUESTION 6 (10 Marks)

(i) Find the point of intersection of the two parabolas $y = 4 - x^2$, $y = 4x - x^2$.

(ii) Write down the equations of the tangents to each of the parabolas at this point.

(iii) If θ is the acute angle between the two tangents in part (ii), find $\tan\theta$.

QUESTION 7 (10 Marks)

A(0, 0); B(1, 0); C(2, 1) are the three vertices of a triangle.

(i) Find the equation of the line BC.

(ii) Find the equation of the line through B perpendicular to AC.

(iii) Find the point F which divides the segment AC internally in the ratio 2:1.

QUESTION 8 (10 Marks)

(i) Sketch the graph of $y = -\frac{1}{2} + \cos x$ for $-\pi \le x \le \pi$.

(ii) Where does the curve cross the x-axis?

(iii) What is the area of the portion of the graph above the x-axis?

QUESTION 9 (10 Marks)

The position of a particle moving along the x-axis is given by $x = -9t + 6t^2 - t^3$ for $t \geq 0$.

(i) What is the velocity at $t = 0$?

(ii) For what values of t does the velocity vanish?

(iii) What is the greatest value of x attained by the particle for $t \geq 0$?

QUESTION 10 (10 Marks)

(i) Find the equation of the parabola with focus (0, 0) and directrix $y = 2$.

(ii) Suppose that the population of a town was 5000 ten years ago and that it increases continuously at a rate proportional to the existing population. If the present population is 7000, what will the population be after a further five years?