N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1968
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Write down the exact values of (a) $\tan\frac{\pi}{4}$ (b) $\sin\frac{11\pi}{6}$ (c) $\cos\frac{5\pi}{4}$

(ii) Find the equation of the line through the point (2, 0) and the intersection of the two lines $x+y+1=0$, $2x-y=3$.

(iii) Find the points (or point) of intersection of the parabola $x^2=4y$ with the line $y=2x-4$.

(iv) Give three inequalities satisfied by every point in the interior of the triangle with vertices (0, 0), (0, 1), (1, 0) and such that no point outside the triangle satisfies all three inequalities.

QUESTION 2 (9 Marks)

(i) Which, if either, of the following quadratic forms is positive definite? (a) $2+4x-x^2$ (b) $2+4x+x^2$

(ii) Find all values of x in the interval $(0, 2\pi)$ which satisfy $(\sin x+2)(2\sin x+1)=0$.

(iii) Write down the derivatives of (a) $\frac{1}{4x^4}$ (b) $\frac{1}{2x+1}$ (c) $\sqrt[4]{x^3}$

QUESTION 3 (9 Marks)

(i) Write down functions whose derivatives are, respectively: (a) $\frac{1}{x^2}$ (b) $\frac{1}{x}$ (c) $\sin x$

(ii) Find the area of the region between $x=0$ and $x=\pi$ bounded by the curve $y=\frac{1}{2}x+\sin x$ and the line $y=\frac{1}{2}x$.

(iii) Find a such that $y=e^{ax}\sin x$ satisfies $\frac{dy}{dx}-2y=e^{ax}\cos x$.

QUESTION 4 (10 Marks)

An urn contains 2 white and 3 red balls. A ball is drawn at random from the urn, then returned to the urn after its colour has been noted. A second ball is drawn at random from the urn. What is the probability that both balls are white? If the first ball is not returned to the urn before the second draw is made, what is the probability that both balls are white?

For each of these experiments what is the probability that 1 white and 1 red ball has been drawn?

QUESTION 5 (10 Marks)

On squared paper, draw the graphs of $y=\log_2 x\ (\frac{1}{2}\le x\le 4)$ and $y=\frac{5}{1+x^2}\ (\frac{1}{2}\le x\le 4)$ using a scale of 2 cm to the unit on both axes.

By addition of ordinates or otherwise, draw the graph of $y=\frac{5}{1+x^2}+\log_2 x\ (\frac{1}{2}\le x\le 4)$.

QUESTION 6 (10 Marks)

Show that the triangle whose sides satisfy $3x+y+1=0$, $x-3y+2=0$, $x+y-1=0$ is right angled. Find the length of the hypotenuse. (Give the answer as a surd with rational denominator.)

QUESTION 7 (10 Marks)

(i) Find the stationary points of $y=x^3-3x^2-9x+30$ and sketch its graph, not on graph paper.

(ii) Find the equations of the tangent and the normal at the point (2, 8).

(iii) Find the coordinates of the points (or point) where the line $y=30-9x$ intersects the curve.

QUESTION 8 (10 Marks)

(i) Find from first principles the gradient of the curve $y=\frac{1}{x}$ at the point (1, 1).

(ii) Find the derivatives of (a) $(1+x^3)^{-3}$ (b) $\log_e\sqrt{\frac{x+1}{x-1}}$

QUESTION 9 (10 Marks)

(i) A particle moves in a straight line with acceleration $\frac{d^2x}{dt^2}=4(2-3t^2)$ for $t\ge 0$. When $t=0$, $x=0$ and $\frac{dx}{dt}=0$. Determine x as a function of t.

(ii) A particle moves along a straight line, its distance from the origin being given as a function of time by $x=\cos^2 t$.

Sketch (not on graph paper) the displacement-time graph for $0\le t\le 2\pi$. Calculate the velocity v and the acceleration f, and show that, for all t, $f=2-4x$.

QUESTION 10 (10 Marks)

(i) Find the derivatives of (a) $\log_e(\sin x)$ (b) $e^{\cos x}$

(ii) Evaluate the following definite integrals correct to 4 decimal places (a) $\int_1^{30}\frac{dx}{x}$ (b) $\int_{\pi/4}^{\pi/2}\cot x\,dx$