N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1967
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (30 Marks)

(i) Draw a diagram showing the region in which both the inequalities $x - 3y > 0, 3x + y > 0$ hold.

(ii) Simplify $\frac{\cos x + \sin x}{\cos x - \sin x} + \frac{\cos x - \sin x}{\cos x + \sin x}$

(iii) P divides AB internally in the ratio 2:1, Q divides AB externally in the same ratio, where A is the point (-2, 1) and B is (1, -5). What are the coordinates of P and Q?

(iv) Write down the derivatives of (a) $5x^{1/5}$ (b) $x\cos x$

(v) Write down primitive functions of (a) $\frac{1}{(2x+1)^2}$ (b) $\frac{1}{2x+1}$

(vi) Find the value of k for which $y = e^{-2x}$ satisfies $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + ky = 0$.

(vii) A number n, $1 \le n \le 299$, is picked at random. What is the probability (expressed as a rational number) that n is divisible by 5?

(viii) A ball is picked at random from a bag containing 2 red and 5 black balls; then a ball from a second bag containing 5 red and 2 black balls. What is the probability (expressed as a rational number) that both balls drawn are red?

(ix) The curve $y = \sqrt{\sin \pi x}$, $0 \le x \le 1$, is rotated about the x-axis. What is the volume of the solid thus generated?

(x) What are the signs of (a) sin 3.1 (b) sin 3.2 (c) cos 4.6?

QUESTION 2 (10 Marks)

(i) R is the foot of the perpendicular from the point P(-5, 10) to the line $4x - 3y = 0$. Find the coordinates of R.

(ii) Find all solutions of $2\cos^2 x - 3\cos x - 2 = 0$ with $-\pi < x < \pi$.

QUESTION 3 (10 Marks)

The sum of the first n terms of an arithmetic sequence is $-n^2 + 21n$.
Find
(i) the first term;

(ii) the n-th term;

(iii) the common difference;

(iv) the first value of n for which the n-th term of the sequence is negative.

QUESTION 4 (10 Marks)

(i) Find from first principles the gradient of the curve $y = 1 - x^2$ at the point (1, 0)

(ii) Find the derivatives of (a) $\log (x + \sqrt{x^2 - 1})$ where $x > 1$ (b) $e^{x - \frac{1}{x}}$

QUESTION 5 (10 Marks)

Find the stationary points of $y = 12 - 12x^2 - 4x^3 + 3x^4$, determine their nature and sketch the curve.

For what values of x is the function (i) monotonically increasing (ii) monotonically decreasing?

QUESTION 6 (10 Marks) {Suitable for 3 Unit Students}

Use the focus - directrix definition to obtain the equation of a parabola with focus $(0, a)$ and directrix $y + a = 0$.

Tangents are drawn from the point $(0, -2a)$ to the parabola. Find the coordinates of their points of contact.

ALTERNATIVE QUESTION 6 (10 Marks) {Suitable for 2 Unit Students}

Use the focus - directrix definition to obtain the equation of a parabola with focus $(0, a)$ and directrix $y + a = 0$.

The latus rectum of the parabola $x^2 = 8y$ cuts the curve in A, B. Find the coordinates of A, B.

The tangents at A, B meet in C, determine the coordinates of C.

QUESTION 7 (10 Marks)

After its engine is cut off, a ship travels in a straight line with a deceleration proportional to its velocity, i.e. $\frac{dv}{dt} = -kv$.

If its velocity is initially 30 cm/s and is 25 cm/s 10 seconds later, when will its velocity be 5 cm/s? (Give your answer to two significant figures.)

QUESTION 8 (10 Marks)

Sketch the graph of $y = 1 + 2\cos x$ for the interval $-\pi \le x \le \pi$.

Find the area under the curve between $x = 0$ and $x = \frac{11\pi}{12}$ (Express the answer in terms of π.)