XIII Asian Pacific Mathematics Olympiad
March, 2001

Time allowed: 4 hours

No calculators to be used

Each question is worth 7 points

Problem 1.

For a positive integer n let $S(n)$ be the sum of digits in the decimal representation of n. Any positive integer obtained by removing several (at least one) digits from the right-hand end of the decimal representation of n is called a *stump* of n. Let $T(n)$ be the sum of all stumps of n. Prove that $n = S(n) + 9T(n)$.

Problem 2.

Find the largest positive integer N so that the number of integers in the set $\{1, 2, \ldots, N\}$ which are divisible by 3 is equal to the number of integers which are divisible by 5 or 7 (or both).

Problem 3.

Let two equal regular n-gons S and T be located in the plane such that their intersection is a $2n$-gon ($n \geq 3$). The sides of the polygon S are coloured in red and the sides of T in blue.

Prove that the sum of the lengths of the blue sides of the polygon $S \cap T$ is equal to the sum of the lengths of its red sides.

Problem 4.

A point in the plane with a cartesian coordinate system is called a *mixed point* if one of its coordinates is rational and the other one is irrational. Find all polynomials with real coefficients such that their graphs do not contain any mixed point.

Problem 5.

Find the greatest integer n, such that there are $n+4$ points A, B, C, D, $X_1, \ldots, X_n$ in the plane with $AB \neq CD$ that satisfy the following condition: for each $i = 1, 2, \ldots, n$ triangles ABX_i and CDX_i are equal.