10th Asian Pacific Mathematics Olympiad

March 1998

Time allowed: 4 hours.
No calculators to be used.
Each question is worth 7 points.

1. Let F be the set of all n-tuples $(A_1, A_2, \ldots, A_n)$ where each A_i, $i = 1, 2, \ldots, n$ is a subset of $\{1, 2, \ldots, 1998\}$. Let $|A|$ denote the number of elements of the set A.
   Find the number $\sum_{(A_1, A_2, \ldots, A_n)} |A_1 \cup A_2 \cup \ldots \cup A_n|$.

2. Show that for any positive integers a and b, $(36a+b)(a+36b)$ cannot be a power of 2.

3. Let a, b, c be positive real numbers. Prove that $\left(1+\frac{a}{b}\right)\left(1+\frac{b}{c}\right)\left(1+\frac{c}{a}\right) \geq 2\left(1+\frac{a+b+c}{\sqrt[3]{abc}}\right)$.

4. Let ABC be a triangle and D the foot of the altitude from A. Let E and F be on a line through D such that AE is perpendicular to BE, AF is perpendicular to CF, and E and F are different from D. Let M and N be the midpoints of the line segments BC and EF, respectively. Prove that AN is perpendicular to NM.

5. Determine the largest of all integers n with the property that n is divisible by all positive integers that are less than $\sqrt[3]{n}$.

END OF PAPER