Question 1 **Marks**

Using a suitable substitution, the definite integral $\int_0^{\frac{\pi}{24}} \tan(2x)\, sec^2(2x)\, dx$ is equivalent to: 1

A. $\frac{1}{2}\int_0^{\frac{\pi}{24}} u\, du$

B. $2\int_0^{\frac{\pi}{24}} u\, du$

C. $\int_0^{2-\sqrt{3}} u\, du$

D. $\frac{1}{2}\int_0^{2-\sqrt{3}} u\, du$

Question 2

The volume of the solid formed by rotating the graph of $y = \sqrt{9-(x-1)^2}$ about the x-axis is given by: 1

A. $4\pi(3)^2$

B. $\pi\int_{-3}^{3}(9-(x-1)^2)\, dx$

C. $\pi\int_{-2}^{4}\sqrt{9-(x-1)^2}\, dx$

D. $\pi\int_{-2}^{4}(9-(x-1)^2)\, dx$

Question 3 **Marks**

The definite integral $\int_{-1}^{1} xe^x\,dx$ is equal to: 1

I. $\int_{-1}^{1} -xe^{-x}\,dx$

II. $2\int_{0}^{1} xe^x\,dx$

III. $\int_{0}^{1} x(e^x - \frac{1}{e^x})\,dx$

A. I and II are true

B. II and III are true

C. I and III are true

D. I, II and III are all true

Question 4

The curve given by $y = sin^{-1}(2x)$ where $0 \leq x \leq \frac{1}{2}$ is rotated about the y-axis to form a solid of revolution. 1

The volume of the solid may be found by evaluating:

A. $\frac{\pi}{4}\int_{0}^{\frac{\pi}{2}}(1 - \cos(2y))\,dy$

B. $\frac{\pi}{8}\int_{0}^{\frac{1}{2}}(1 - \cos(2y))\,dy$

C. $\frac{\pi}{8}\int_{0}^{\frac{\pi}{2}}(1 - \cos(2y))\,dy$

D. $\frac{1}{8}\int_{0}^{\frac{\pi}{2}}(1 - \cos(2y))\,dy$

Question 5 (Start on a New page) (14 marks) **Marks**

a) Find $\int \sin(x/3)\cos(x/2)\,dx$, 2

b) Use integration by parts to evaluate $\int_0^{\frac{\pi}{4}} x\,sec^2x\,dx$ 2

c)

i). Find the values of the constants a and b such that 2

$$\frac{5-x}{(2x+3)(x^2+1)} = \frac{a}{2x+3} - \frac{bx-1}{x^2+1}$$

ii). Hence find $\int \frac{5-x}{(2x+3)(x^2+1)}dx$ 2

d) Find $\int \frac{\sqrt{9+4x^2}}{x}dx$ 3

e) Use the substitution $u = \frac{1}{x} - x$ to evaluate $\int \frac{1+x^2}{1+x^4}dx$ 3

End of Question 5

Question 6 (Start on a New Page) (16 Marks) **Marks**

a) The region bounded by the curve $y^2 = x - 1$ and the line $x = 3$ is rotated about the line $y = 3$.

i). Draw a typical slice and mark in the radius and the width of the typical slice. 2

ii). Write down an expression for the volume of the typical slice. 2

iii). Find the volume of the solid. 2

b) The area bounded by the curve $y = 2cos\ x$ for $0 < x \leq 2\pi$ and the line $y = 2$ is rotated about the y-axis.

i). Draw a diagram and show a typical cylindrical shell. 1

ii). Show that the volume of a typical cylindrical shell δV, of thickness δx is given by 2

$$\delta V = 4\pi x(1 - \cos x)\delta x$$

iii). Hence find the volume of the solid formed. 2

c) A cake tray has a rectangular base with sides 16cm by 12 cm. The top is also rectangular with sides 20 cm by14 cm. The tray has a depth of 5 cm.

i). Show that the volume of a slice of thickness δh at height h from the base of the tin is given by 3

$$\delta V = \frac{8}{25}(h^2 + 50h + 600)\delta h$$

ii). Hence find the volume of the cake tray. 2

End of Question 6