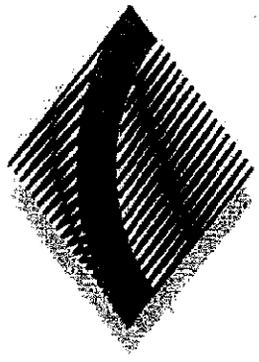
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Name: _____

Class: 12MTZ1

Teacher: _____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2008

YEAR 12

AP3 EXAMINATION

MATHEMATICS EXTENSION 2

Time allowed - $1\frac{1}{2}$ HOURS

(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- The marks to be awarded for each question are indicated on the examination paper.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used and standard integral tables are provided.
- Your solutions will be collected in one bundle stapled in the top left hand corner. Please arrange them in order Q1 to 4.

***** Each page must show your name and your class. *****

Question 1 (15 Marks) **Marks**

(a) Two complex numbers w and z are such that

$$w+z=6+2i,$$
$$w-3z=\frac{20}{2-i}$$

Find w and z, giving each answer in the form $x+iy$. **2**

(b) The points P and Q represent the complex numbers p and q respectively, where

$$p=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4},$$
$$q=2\cos\frac{\pi}{4}+2i\sin\frac{\pi}{4}.$$

Find the moduli of the complex numbers a, b and c where

(i) $a=p+\overline{p}$ **1**

(ii) $b=\frac{1}{q}$ **1**

(iii) $c=q^2$ **1**

(c) Find the argument (in the interval $-\pi\le\theta\le\pi$) of $(1+\sqrt{3}i)^{20}$, giving your answer as an exact value. **3**

(d) (i) Given that $(x+iy)^2=-5+12i$, where x and y are real, find the set of possible values of $x+iy$. **3**

(ii) By completion of square, or otherwise, solve $z^2+4z=-9+12i$. **2**

(e) The complex number z is given by $z=\cos\theta+i\sin\theta$,
Show that $|1+z|^2=2(1+\cos\theta)$. **2**

Question 2 (17 Marks) BEGIN A NEW PAGE **Marks**

(a) For $P(x) = x^5 - 3x^4 + 2x^3 - 2x^2 + x + 1$,

(i) Show that $(x^2 + 1)$ is a factor of $P(x)$ using the Factor Theorem. **1**

(ii) Given that $Q(x) = x^3 - 3x^2 + x + 1$ is a factor of $P(x)$, factorise $P(x)$ completely. **1**

(iii) Hence find all its real and complex zeros. **2**

(b) Prove that if $P(x) = (x-\mu)^r S(x)$, where $r \geq 1$ and $S(\mu) \neq 0$, then $P'(x)$ has a root μ of multiplicity $(r-1)$. **2**

(c) (i) For what values of c is $z - ci$ a factor of $P(z) = z^4 - z^3 + 9z^2 - 4z + 20$? **2**

(ii) Hence, solve $P(z) = 0, c \neq 0$. **2**

(d) If α, β and γ are roots of $2x^3 + 3x^2 + x - 5 = 0$, find the value of

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **1**

(ii) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma}$ **1**

(iii) $\dfrac{1}{\alpha\beta\gamma}$ **1**

(iv) Hence, form the polynomial whose roots are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$. **1**

(e) Express $\dfrac{x^2 + x + 2}{(x-4)(x^2+1)}$ as a sum of partial fractions. **3**

Question 3 (12 Marks) BEGIN A NEW PAGE **Marks**

(a) Consider the graph of the function $f(x)=\dfrac{(x-2)^2}{x-1}$.

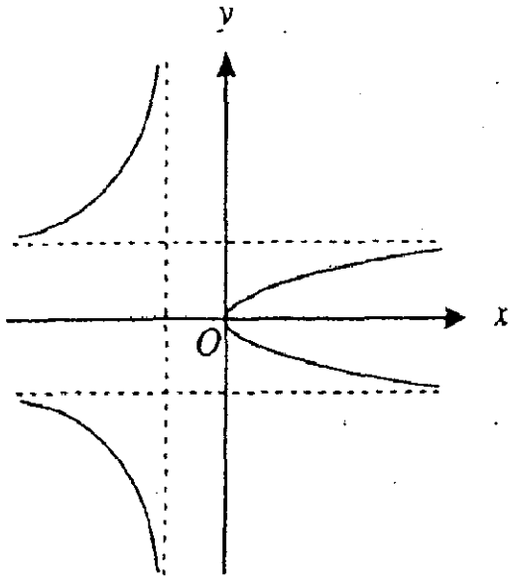
(i) Show that $y=x-3$ is an asymptote of $f(x)$. **1**

(ii) Determine the behaviour of $f(x)$ as $x\to\infty$ and $x\to-\infty$, and as x approaches other critical value(s). **2**

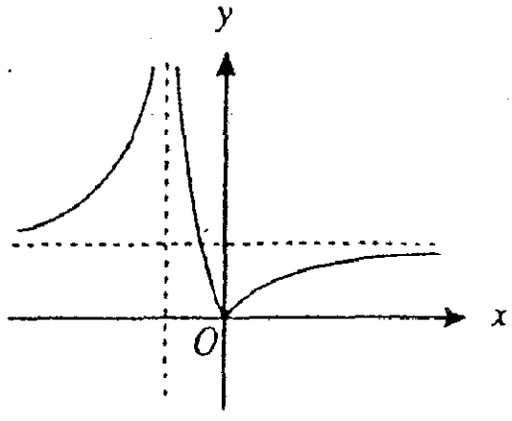
(iii) Hence, sketch the graph of $f(x)$ on the number plane, labelling clearly its asymptotes, x and y intercept(s). **2**

(iv) Hence, sketch the graph of $y=\sqrt{f(x)}$ on the same number plane. **1**

(b) Given the graph of $y^2=f(x)$ and $y=|f(x)|$ as shown.



$y^2=f(x)$



$y=|f(x)|$

(i) Explain the shape and position of $y=f(x)$ in relation to the above graphs. **2**

(ii) Hence, sketch the graph of $y=f(x)$ **2**

(iii) Hence, sketch the graph of $y=f(|x|)$ **2**

Question 4 (18 Marks) BEGIN A NEW PAGE **Marks**

(a) A hyperbola has the equation $\frac{x^2}{16}-\frac{y^2}{9}=1$,

(i) Sketch the graph of this hyperbola, labelling clearly its vertices, foci and asymptotes. **2**

(ii) Two tangents from P and Q on the hyperbola meet at the point $(-1,-2)$, Write down the equation of the chord of contact PQ. **1**

(b) The equation of an ellipse is $\frac{(x-1)^2}{9}+\frac{y^2}{4}=1$

(i) Show that the equation of the tangent to the ellipse at $\left(2,\frac{4\sqrt{2}}{3}\right)$ is $x+3\sqrt{2}y-10=0$. **3**

(ii) Find the coordinates of the point of intersection of the tangent with one of the directrices of the ellipse in exact form. **2**

(c) The parametric equations of a hyperbola are

$x=\sec\theta,\quad y=4\tan\theta \qquad \theta\neq\frac{-\pi}{2},\frac{\pi}{2}$

(i) Write the Cartesian equation of the hyperbola. **1**

(ii) Show that the gradient of the tangent to the hyperbola at any point $\theta\neq 0$ is $4\operatorname{cosec}\theta$. **2**

(iii) Hence find the equation of the normal at $\theta=\frac{\pi}{4}$. **2**

(d) The equation of a rectangular hyperbola is $xy=1$.

(i) Given the equation of the normal at P, where $x=t$ is $ty-1=t^3x-t^4$, find the coordinates of Q where the normal intersect the hyperbola again in terms of t. **3**

(ii) Hence show that the equation of the locus of the midpoint of PQ is $4xy=2-\left(\frac{t^8+1}{t^4}\right)$. **2**

The End