YEAR 11 ASSESSMENT 1995
TERM 4 - 4 UNIT

Time 85 minutes.

All questions are to be attempted.
Each question to be handed in separately.

QUESTION 1. (Start a new page)

(a) Express in the form $a + ib$, where a and b are real:

(i) $(2-3i)^2$ (ii) $\sqrt{-2i}$

(b) z is the complex number $1+i$.

(i) Write in mod-arg form;

(α) z

(β) iz

(γ) $\frac{1}{z}$

(ii) Show on an Argand diagram, the points A, B and C, representing respectively z, iz and $\frac{1}{z}$, clearly showing the relations between their mods and args.

(c) (i) Show that the equation:

$$|z+1| = 2|z-2|$$

represents a circle on the Argand Diagram, and find its centre and radius.

(ii) By means of a diagram drawn roughly to scale, explain, giving all possible information, how arg z and mod z vary as the point corresponding to z moves around the circle.

QUESTION 2 (Start a new page)

(a) Express in mod-arg form:

(i) $\frac{r(\cos\theta + i\sin\theta)}{s(\cos\theta - i\sin\theta)}$, where r and s are real numbers.

(ii) z^5 if $z = \sqrt{3}(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

(b) (i) Find the three cube roots of unity.

(ii) Show that, if ω is one of the non-real roots, then the other non-real root is ω^2.

(iii) Evaluate $(1+\omega)(1+\omega^2)$.

(c) Three points A_1, A_2 and A_3 represent the three cube roots of a complex number of modulus one on an Argand diagram. If point P represents the complex number z and $|z| = 3$, show that:

$$PA_1^2 + PA_2^2 + PA_3^2 = 30.$$