HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS – EXTENSION 2

2013 HSC

ASSESSMENT TASK 1

December 2012

Examiners ~ G Huxley, S Gee

INSTRUCTIONS:

- Reading Time – 3 minutes.
- Working Time – 40 minutes.
- Attempt **all** questions.
- **Section I ~**Answer multiple choice questions on the answer sheet provided.
- **Section II ~ Each question is to be answered in the booklets provided. All** necessary working should be shown.
- This examination paper must **NOT** be removed from the examination room.
- This paper contains 4 multiple choice questions worth 1 mark each and two free response questions worth 13 marks each.
- Marks may **not** be awarded for careless or badly arranged work.
- Board approved calculators may be used. Use of mathematical templates is optional.

STUDENT NAME: _____

TEACHER: _____

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SECTION I 4 marks

Attempt Questions 1 - 4
Use the multiple choice answer sheet provided.

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1 Given the coordinates of S are $(0,6)$ and S' is $(0,-6)$ and the conic obeys the condition $|PS| + |PS'| = 20$.

The length of the semi major axis is

(A) 20 (B) 16 (C) 10 (D) 8

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2 The equation of the conic whose distance from the point (1, 0) is half its distance from the line $x = 4$ is given by:

(A) $\frac{x^2}{4} + \frac{y^2}{3} = 1$ (B) $\frac{x^2}{3} + \frac{y^2}{4} = 1$

(C) $\frac{x^2}{4} - \frac{y^2}{3} = 1$ (D) $\frac{x^2}{3} - \frac{y^2}{4} = 1$

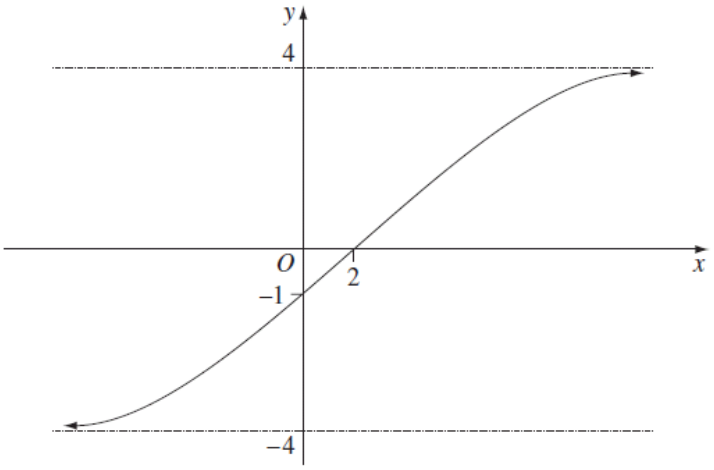
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3. Which of the following is an odd function?

(A) $y = \frac{e^x + e^{-x}}{2}$ (B) $y = x\sin x$

(C) $y = \frac{e^x - e^{-x}}{2}$ (D) $y = 3x^5 - 2x^3 - 1$

4.



The graph above shows the function $y=f(x)$.
For the function defined by $y=\sqrt{f(x)}$ the range is given by

(A) $-4 < y < 4$ (B) $0 \le y < 4$

(C) $-2 < y < 2$ (D) $0 \le y < 2$

SECTION 2 26 marks

Attempt Questions 5 – 6. Show all necessary working.

Question 5: 13 marks Start a new booklet

NOTE: ALL GRAPHS SHOULD BE AT LEAST ONE THIRD OF A PAGE AND DRAWN IN PENCIL.

			Marks
(a)	(i)	Sketch $y = \lvert x \rvert - 1$	**1**
	(ii)	Sketch $y = x\left(\lvert x \rvert - 1\right)$	**1**
(b)	(i)	Differentiate $\sqrt{x} + \sqrt{y} = 1$ implicitly.	**1**
	(ii)	Determine the domain and range of $\sqrt{x} + \sqrt{y} = 1$	**1**
	(iii)	Comment on the nature of any critical points.	**1**
	(iv)	Sketch $\sqrt{x} + \sqrt{y} = 1$	**1**
	(v)	Sketch $\sqrt{\lvert x \rvert} + \sqrt{\lvert y \rvert} = 1$	**1**
(c)	(i)	On the same set of axes sketch the curves $f(x) = x^2$ and $g(x) = \frac{1}{x}$ and hence $R(x) = f(x) + g(x)$	**3**
	(ii)	On separate sets of axes, sketch	
		$(\propto)$ $y = \lvert R(x) \rvert$	**1**
		(β) $y = R'(x)$	**1**
		(γ) $y = \left[R(x)\right]^{-1}$	**1**

Question 6: 13 marks Start a new booklet

Marks

(a) Determine the real values of λ for which the equation

$\frac{x^2}{4-\lambda} + \frac{y^2}{2-\lambda} = 1$ defines

(α) an ellipse **1**

(β) a hyperbola **1**

(b) The hyperbola $\mathcal{H}$ has the equation $9y^2 - 4x^2 = 36$.

(i) Write down:

(α) its eccentricity **1**

(β) the coordinates of its foci S and S' **1**

(γ) the equation of each directrix **1**

(δ) the equation of each asymptote. **1**

(ii) Sketch the hyperbola $\mathcal{H}$. Show any intercepts as well as the features found in part (i) above. **1**

(c) For the ellipse: $E = 5x^2 + 9y^2 = 45$.

i) Find the equation of the tangent at the point $P\left(2, \frac{5}{3}\right)$ **2**

ii) Find the equation of the normal at the point $P\left(2, \frac{5}{3}\right)$ **1**

iii) The normal meets the major axis in G ,and
OQ is the perpendicular from the centre O of the ellipse E
to the tangent at P.
Draw a diagram to represent this information.
Show that the value of $|PG| \times |OQ|$
is equal to the square of the semi-minor axis. **3**

NAME/STUDENT NUMBER:…… ……..

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ASSESSMENT TEST 1

SECTION 1 ANSWERS

1	(A)	(B)	(C)	(D)
2	(A)	(B)	(C)	(D)
3	(A)	(B)	(C)	(D)
4	(A)	(B)	(C)	(D)