QUESTION ONE (Start a new answer booklet)

(a) (i) Solve the quadratic equation $2x^2 + 3x - 2 = 0$.

(ii) Sketch the parabola $y = 2x^2 + 3x - 2$, showing the zeroes, the y intercept and the vertex.

(iii) Solve the quadratic inequation $2x^2 + 3x - 2 > 0$.

(b) If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$, find the value of:

(i) $\alpha + \beta$,

(ii) $\alpha\beta$,

(iii) $\alpha^2 + \beta^2$.

(c) Given $4x^2 + 12x + b \equiv (ax + 3)^2$ for all x, find the values of a and b.

(d) For what values of k will the quadratic equation $x^2 + 3x + k = 0$ have:

(i) equal roots.

(ii) no real roots.

(iii) one root the reciprocal of the other.

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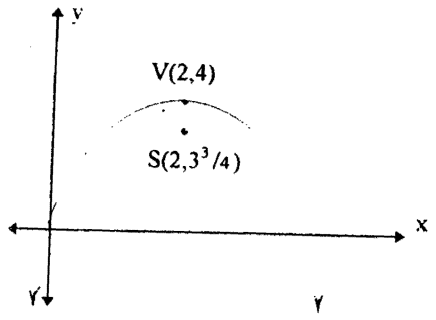
QUESTION TWO (Start a new answer booklet)

(a) (i) For the parabola $x^2 = 8(y-3)$. find the co-ordinates of the focus and the equation of the directrix.

(ii) By completing squares. find the centre and radius of the circle:

$$x^2 - 2x + y^2 + 4y - 11 = 0.$$

(b)



The diagram above shows a parabola with vertex $V(2.4)$ and focus $S(2.3\frac{3}{4})$. Find the equation of the parabola.

(c) Two integers have a sum of 12. Let the integers be x and $12 - x$.

(i) Write down an expression for the product of one of the numbers with the square of the other.

(ii) Find the two integers for this expression to be a maximum.

(d) Given the points $A(0.0)$ and $B(3.0)$. a point $P(x.y)$ moves so that the distance PA is always twice the distance PB. Prove that the locus of P is a circle. centre (4.0) and radius two units.

Exam continues overleaf ...

QUESTION THREE (Start a new answer booklet)

(a) Consider the function $f(x) = x^3 - 6x^2 + 8$.

(i) Find the stationary points and their nature.

(ii) Find any point of inflexion.

(iii) Sketch the curve. (You need not find the x-intercepts.)

(b) The height h metres of a cricket ball t seconds after being struck is given by:

$$h = 98t - 9{\cdot}8t^2.$$

(i) Find the greatest height reached by the ball.

(ii) Find the total time the ball is in the air.

(c) (i) What is the domain of the function $y = \log_e(x + 1)$?

(ii) Sketch the curve $y = \log_e(x + 1)$ for $-1 < x \leq 3$.

(iii) The volume of the solid of revolution formed when the section of the curve $y = \log_e(x + 1)$ from $x = 0$ to $x = 2$ is rotated around the x-axis is given by

$$V = \pi \int_0^2 \Big(\log_e(x + 1)\Big)^2 \, dx.$$

Use Simpson's rule with three function values to approximate this integral. (Leave your answer correct to two decimal places.)

Exam continues next page ...

QUESTION FOUR (Start a new answer booklet)

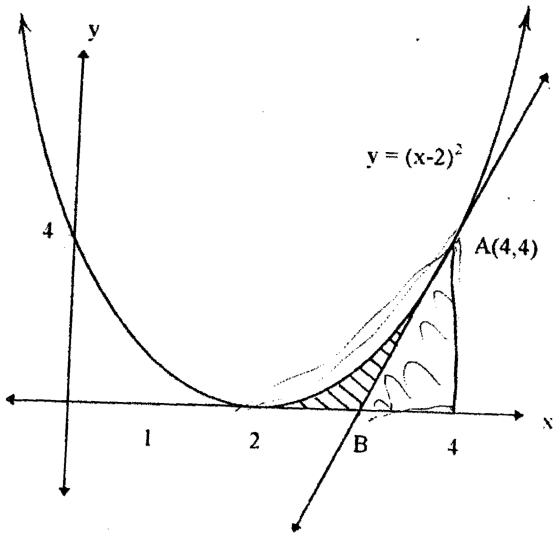
(a) Find the following indefinite integrals:

(i) $\int \sqrt{x}\, dx$,

(ii) $\int (t+3)^2\, dt$.

(iii) $\int \dfrac{2x^2+3}{x}\, dx$.

(b)



The diagram above shows the parabola $y = (x-2)^2$.

(i) Find the equation of the tangent to the curve $y = (x-2)^2$ at the point $(4, 4)$.

(ii) Show that this tangent crosses the x-axis at the point $B(3, 0)$.

(iii) Find the area of the shaded region bounded by the parabola, the tangent and the x-axis.

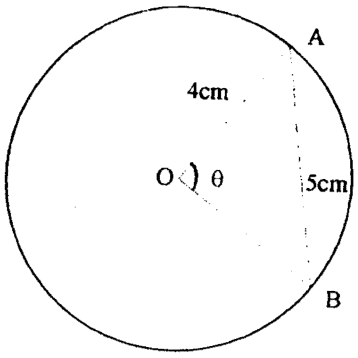
(c) The point $(1, 1)$ is a stationary point on the curve $y = f(x)$. Find the equation of the curve, given $f''(x) = 2x - 3$.

Exam continues overleaf ...

QUESTION FIVE (Start a new answer booklet)

(a) (i) Express $0{\cdot}8\pi$ radians in degrees.

(ii) Solve $\tan\theta - \sqrt{3} = 0$, for $0 \leq x \leq 2\pi$.



(b) The circle in the diagram above has radius OA of 4cm. A chord AB of length 5cm subtends an angle θ at the centre O.

(i) Use the cosine rule to show that $\cos\theta = \frac{7}{32}$, and hence find the value of θ in radians. (Give your answer to five decimal places.)

(ii) Find the length of the minor arc AB.(Give your answer to two decimal places.)

(iii) Calculate the area of the minor sector OAB. (Give your answer correct to two decimal places.)

(c) (i) State the period and the range of the function $f(x) = 2\sin\frac{x}{2}$.

(ii) Sketch the curve $y = 2\sin\frac{x}{2}$, for $0 \leq x \leq 4\pi$.

(iii) Solve the equation $2\sin\frac{x}{2} = 1$, for $0 \leq x \leq 4\pi$.

Exam continues next page ...

QUESTION SIX (Start a new answer booklet)

(a) Find $\frac{dy}{dx}$ in each of the following:

(i) $y = \ln(x^2 + 1)$,

(ii) $y = e^{\ln x}$.

(b) Prove that the area under the curve $y = \frac{1}{x}$ over the interval $a \leq x \leq b$ where $a > 0$. is the same as the area over the interval $2a \leq x \leq 2b$.

(c) Evaluate the following definite integrals:

(i) $\int_0^2 6e^{3x}\,dx$.

(ii) $\int_0^1 \frac{3x^2}{x^3+1}\,dx$.

(d) (i) Show that $\frac{d}{dx}(x \ln x) = 1 + \ln x$.

(ii) Hence find $\int \ln x\,dx$.

QUESTION SEVEN (Start a new answer booklet)

(a) Find $\frac{dy}{dx}$ given:

(i) $y = x^2 e^{-x}$.

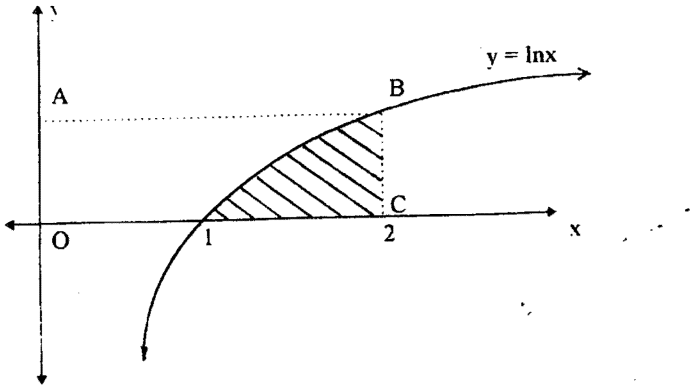
(ii) $y = \sqrt{e^x}$.

(b) (i) On the same set of axes, sketch the curves $y = e^x$ and $y = e^{-x}$ for the domain $-2 \leq x \leq 2$.

(ii) Find the gradients of the tangents to the curves at the point $(0, 1)$. Hence find the angle between the tangents at this point.

Exam continues overleaf ...

(c)



(i) Find an expression for the area of rectangle $OABC$.

(ii) Calculate the area bounded by the curve $y = \ln x$, both axes and the line $y = \ln 2$.

(iii) Hence or otherwise show the area of the shaded region is equal to $2\ln 2 - 1$ square units.

KWM