ASSESSMENT TASK # 2 - April 2000

# MATHEMATICS

## 3/4 UNIT COMMON

*Time allowed — One and a half hours*
*(Plus 5 minutes reading time)*

*Examiners: F. Jordan, P. Bigelow*

**DIRECTIONS TO CANDIDATES**

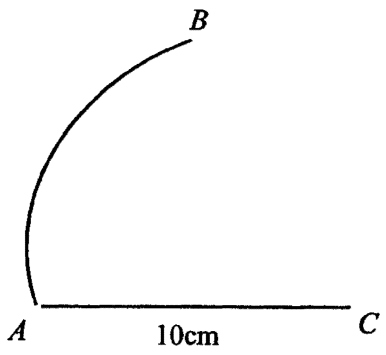
- *All* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- There are **6** questions and *each question* is to be returned in a *separate* booklet, clearly marked Question 1, etc. Each booklet must also show your name and teacher.
- Start each question in a new booklet.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

## Section A
## START A NEW BOOKLET

**Question 1.** (8 marks)

**Marks**

(a) Find the perimeter of $ABC$, where $AB$ and $BC$ are circular arcs of radius 10cm, centred at $C$ and $A$ respectively. **2**




(b) Evaluate $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta}$. **2**

(c) Differentiate (i) $e^{2x^3}$ **2**

(ii) $\ln\sqrt{x}$

(d) State the values of $x$ for which $y = x^3 - 3x$ is a decreasing function.

**Question 2** (8 marks) **(Start a new page.)**

**Marks**

(a) Differentiate (i) $\cos x$ **4**

(ii) $\sin^3 x$

(iii) $2\tan\left(\frac{x}{2}\right)$

(b) Find a primitive of: **4**

(i) $\cos 2x$

(ii) $x + \frac{1}{x}$

(iii) $\frac{2x}{x^2 - 1}$

## Section B
## START A NEW BOOKLET

**Question 3.** (8 marks)

| | | Marks |
|---|---|---|
| (a) | Evaluate $\int_0^{\frac{1}{4}} \sec^2 \pi x$ | 2 |
| (b) | Find the area bounded by the $x$-axis, the ordinates $x = -1$, $x = 1$, and the curve $y = \frac{1}{2}\left(e^x + e^{-x}\right)$ | 2 |
| (c) | Find the stationary points of the curve $y = x\left(x^2 - 1\right)$, and determine their nature. | 2 |
| (d) | If $f'(x) = \frac{1}{\sqrt{x}}$, find an expression for $f(x)$, given that $f(0) = 1$. | 2 |

**Question 4** (8 Marks) **(Start a new page.)**

| | | Marks |
|---|---|---|
| (a) | An open tank is to be made of sheet iron; it must have a square base and sides perpendicular to the base. Its capacity is to be $8m^3$.<br>Find, correct to the nearest centimetre, the side of the square base, and the depth, so that the least amount of sheet iron may be used. | 5 |
| (b) | Find the area of the minor segment cut from a circle of radius 10cm by a chord of length 12cm. Give your answer correct to 4 significant figures. | 3 |

## Section C
**START A NEW BOOKLET**

**Question 5:** (8 marks)

| | | Marks |
|---|---|---|
| (a) | Find the derivative of $y = \ln\sqrt{\frac{1-x}{1+x}}$. | 2 |
| (b) | Write down the equation of the tangent to the curve $y = e^{x^2}$ at the point where $x = 1$. | 2 |
| (c) | Use mathematical induction to prove that for all positive integers $n$: $1+2+4+\ldots+2^{n-1} = 2^n - 1$. | |

**Question 6:** (8 marks) **(Start a new page.)**

| | | Marks |
|---|---|---|
| (a) | On the same axes sketch the graphs of the curves $y = \cos x$ and $y = -\sin x$ in the domain $0 \le x \le 2\pi$. | 2 |
| (b) | Find the points of intersection of the two curves in Part (a). | 2 |
| (c) | By use of the integral calculus, find the area bounded by the curves $y = \cos x$ and $y = -\sin x$ in the region where $0 \le x \le 2\pi$. | 4 |

(Start a new booklet) Marks

(a) Sketch $y = \tan x$ for $|x| < \frac{\pi}{2}$. 2

By sketching the appropriate line find the number of roots of the equation $\tan x - x = \frac{\pi}{2}$ within this domain.

(b) (i) Show that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ 5

(ii) Hence, using induction or otherwise prove that $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$

(c) (i) Find any stationary point and points of inflexion on the curve defined below for $x > 0$. 4

$$y = x^2 \ln\left(\frac{1}{x^3}\right)$$

(ii) Sketch the curve.

**END OF THE PAPER**