SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

3/4 UNIT COMMON

APRIL 1999

MATHEMATICS

Time allowed: *90 Minutes*
(plus 5 minutes reading time)

Examiners: R Boros, B.Genner.

Directions to Candidates

- All questions may be attempted.
- NOT all questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question on a new page, clearly showing your name and class.
- If required, additional paper may be obtained from the Examination Supervisor upon request.

Question 1 **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{\sin 7x}{6x}$ **2**

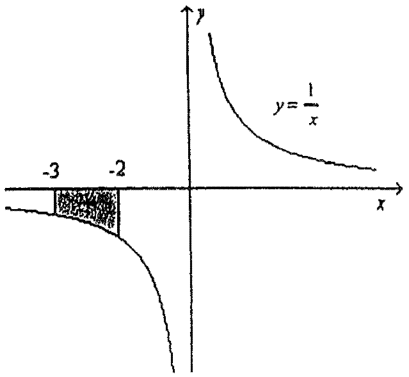
(b) Differentiate: **3**

(i) $\cos 2x$

(ii) $x \tan x$

(c) Find $\int_0^{\frac{\pi}{7}} \sin \frac{x}{2} \, dx$ **2**

(d) **2**



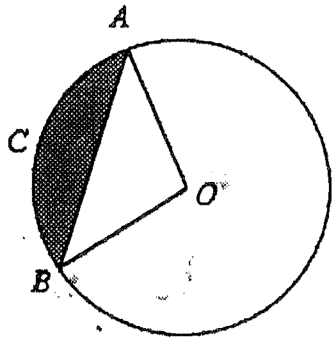
Find the value of the shaded area correct to 2 decimal places

Question 2 (Start a new booklet) Marks

(a) Differentiate $\frac{x}{\ln x}$ 2

(b) Find a primitive of $\frac{2}{4+3x}$ 2

(c) 3



A circle has centre O and radius 10 cm. $\angle AOB = \frac{5\pi}{6}$

Find the exact value of the shaded region.

(d) If $y = x^n e^{ax}$ show that $\frac{dy}{dx} - ay = \frac{ny}{x}$ 2

. **Question 3 (Start a new booklet)** **Marks**

(a) (i) For the curve $y = 2\sin(\frac{\pi}{2} + 4x)$ state the period and amplitude **4**

(ii) Sketch $y = 2\sin(\frac{\pi}{2} + 4x)$ for $-0 \cdot 5\pi \leq x \leq 0 \cdot 5\pi$

(b) Find $\frac{d}{dx}\{\log_e(\cos x)\}$ and hence find the area under the curve $y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$ **3**

(c) The curve $y = 3 + 2\sin x$ is rotated about the x axis between $x = 0$ and $x = \pi$. Find the volume of the solid generated. **3**

Question 4 (Start a new booklet) Marks

(a) State the domain and range of $y = \sqrt{1 + \ln x}$ 2

(b) Find $\int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)dx$ 2

(c) Use one step of Newton's Method to find an improved value of that root of $f(x) = e^{4x} - \sqrt{x + 0 \cdot 81}$ which lies close to $x = 0$ 3

(d) The planet Hollywood is in a far, far away galaxy and has a moon with diameter of 148 000 km. If the distance between the centre of Hollywood and the centre of the moon is $1 \cdot 28 \times 10^7$ km, find the angle subtended by the moon at the centre of the planet in seconds. 2

Question 5 (Start a new booklet) **Marks**

(a) Find the equation of the normal to $y = 3\cos\frac{x}{2}$ at the point where $x = \frac{\pi}{3}$ **3**

(b) The *error function* is given by $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$. **3**

Use the trapezoidal rule with 5 function values to estimate erf(2)

(c) If $f(x) = 2e^{-x^2}$ then **3**

(i) Find $f(\sqrt{\ln x})$

(ii) Find a, where $f''(a) = 0$

(d) Find the area enclosed by $f(x) = 3(x-2)$ and $g(x) = (x-1)(x-2)(x-3)$. **3**

[HINT: Make a sketch]