SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

APRIL 1996 ASSESSMENT TASK

MATHEMATICS

3/4 UNIT COMMON

Time allowed - 90 minutes
(Plus 5 minutes reading time)

Examiner: PS Parker

DIRECTIONS TO CANDIDATES

- ALL questions may be attempted.
- **NOT** all questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question on a new page, clearly showing your name and class.
- If required, additional paper may be obtained from the Examination Supervisor upon request.

Question 1 (Start a new page) **Marks**

(a) Differentiate: **8**

(i) $ln(x^2 - 3)$

(ii) $\sin 2x$

(iii) $x \tan 2x$

(iv) $\dfrac{e^x}{\cos x}$

(b) Solve for x: $2\ ln(3x - 1) = ln(4)$ **2**

(c) Solve for x: $e^{2x} - 3e^x = 4$ (leave your answer as an exact value) **2**

(d) Calculate $\log_5 12$ correct to 3 decimal places. **2**

Question 2 **(Start a new page)** **Marks**

(a) Write down a primitive function of: **6**

(i) $\cos 3x$

(ii) $\dfrac{1}{1+x}$

(iii) $6e^{x}$

(b) Evaluate the following: **8**

(i) $\displaystyle\int \frac{6x\,dx}{9+x^{2}}$

(ii) $\displaystyle\int \frac{\sin x}{\cos x}\,dx$

(iii) $\displaystyle\int_{-2}^{-1} \frac{dx}{2x-3}$ (leave your answer in exact form)

(c) (i) Find $\dfrac{d(xe^{x})}{dx}$ **1**

(ii) Hence evaluate $\displaystyle\int_{0}^{1} 3xe^{x}\,dx$ **1**

Question 3 (Start a new page) **Marks**

(a) Find the area enclosed between the two curves, $y = x^2 - 5$ and $y = x + 1$. 3

(b) Find the volume of the solid formed when the curve $y = \sqrt{x}$ is rotated about the y axis between $y = 1$ to $y = 4$ 3

(c) Given the function $f(x) = (x^2 + 1)e^x$ 4

(i) Show that $f''(x) = e^x(x+1)(x+3)$

(ii) Hence or otherwise, find any inflexion points. Be sure to justify their existence.

(e) (i) Show that $\dfrac{1}{N(k-N)} = \dfrac{1}{kN} + \dfrac{1}{k(k-N)}$ 2

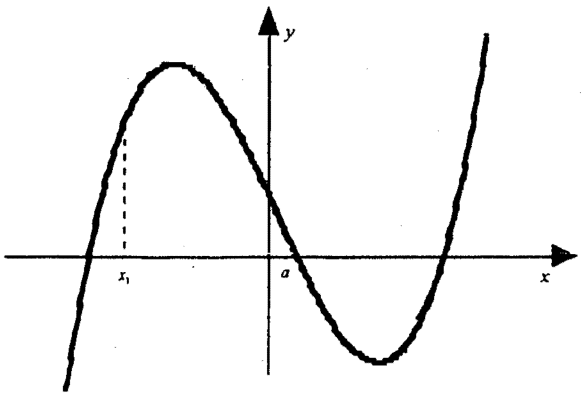
(ii) Hence or otherwise, deduce that the integral $= \frac{1}{k}\ln 2$. 2

$$\int_{\frac{k}{3}}^{\frac{k}{2}} \frac{1}{N(k-N)}\, dN$$

Question 4 (Start a new page) **Marks**

(a) (i) Find an approximation to the root of $y = x^3 + x - 1$ by using Newton's method once and starting with $x = 0{\cdot}6$ as an approximation. **2**

(ii) With the aid of the diagram below, explain why the approximation of $x = x_1$ is not a good choice as a starting point to find the root $x = a$. You should copy this diagram to your answer sheet. **2**



(b) Given $y = x + e^{-x}$ **10**

(i) Find the co-ordinates and the nature of any stationary points on the graph of $y = f(x)$.

(ii) Find $f''(x)$ and show that the graph is concave up for all values of x.

(iii) Describe what happens to y as

(α) $x \to \infty$ (β) $x \to -\infty$

(ii) Sketch clearly, showing co-ordinates of turning points and equations of asymptotes.

Question 5 **(Start a new page)** **Marks**

(a) (i) Prove that $8\cos^4 x = 3 + 4\cos 2x + \cos 4x$. **3**

[Hint $\cos 2\theta = 2\cos^2 \theta - 1$]

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$ **2**

(b) The graph of the function $y = ln\,(x - 1)$ meets the line $y = 2$ at P and the x axis at Q. **7**
From P, perpendiculars are drawn to the x axis and the y axis, meeting at R and S respectively.

(i) Sketch a diagram representing the above information.

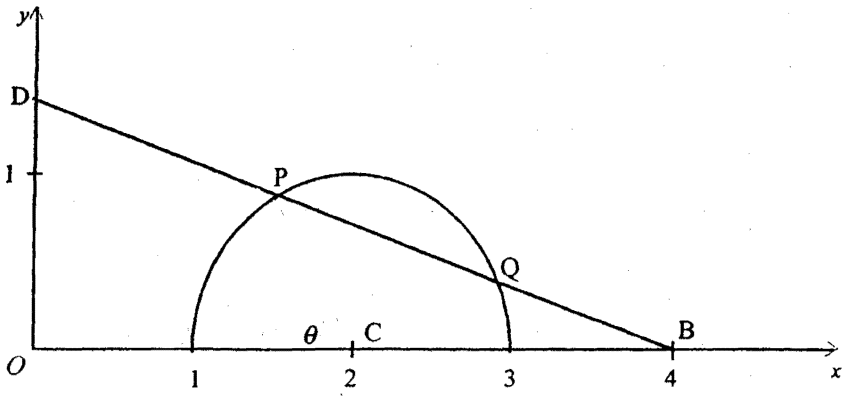
(ii) For $y = ln\,(x - 1)$, make x the subject.

(iii) Hence or otherwise, show that the arc QP, of the graph representing $y = ln\,(x - 1)$, divides the rectangle OSPR into two portions of equal area, where O is the origin.

(iv) Show that the area enclosed between the arc QP and the straight line interval QP equals the area of triangle OSQ.

Question 6 (Start a new page) **Marks**

(a) The diagram shows a semi-circle with centre (2,0) and a radius of one unit lying above the positive x axis. A line from B(4,0) cuts the semi-circle at Q and P, and the y axis at D. Let PCO = θ radians, and let length OD = h units. **10**



(i) Copy the diagram to your answer sheet

(ii) By dropping a perpendicular from P to Ox and considering similar triangles, or otherwise, show that

$$h = \frac{4\sin\theta}{2+\cos\theta}$$

(iii) Find, θ_1, the value of θ for which $\frac{dh}{d\theta} = 0$

(iii) Show that h is a maximum when $\theta = \theta_1$

(b) Evaluate $\int \frac{dx}{x\ln x}$ **3**

END OF PAPER