2014
Task 2
Examination

Number: _____

Teacher: _____

Year 12 Extension 1 Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Section I ~ Pages 1-3

- 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II ~ Pages 4-8

- 60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Teachers:

Mr Bradford (Examiner)
Mr Sedgman
Mr Johansen
Ms Tran

Write your Board of Studies Student Number on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 82

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 The polynomial $x^3 - 4x^2 + x + 6$ factorises to:

(A) $(x+1)(x-2)(x+3)$

(B) $(x-1)(x+2)(x+3)$

(C) $(x+1)(x-2)(x-3)$

(D) $(x-1)(x+2)(x-3)$

2 The equation of the tangent at $T\left(2at, at^2\right)$ to the parabola $x^2 = 4ay$ is:

(A) $y = tx - at^2$

(B) $y = t^2x - a$

(C) $y = tx + at^2$

(D) None of the above

3 The acute angle between the lines $x - y + 1 = 0$ and $\sqrt{2}x - y = 0$ is approximately:

(A) $80°16'$

(B) $9°44'$

(C) $30°$

(D) $53°8'$

4 The solution to $\frac{1}{\sin x} > 2$ on the interval $0 < x < 2\pi$ is:

(A) $0 < x < \frac{\pi}{6} \text{ or } \frac{5\pi}{6} < x < 2\pi$

(B) $0 < x < \frac{\pi}{6}$

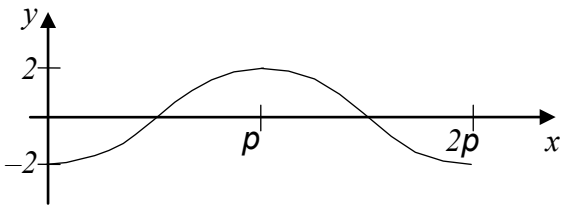
(C) $\frac{5\pi}{6} < x < \pi$

(D) None of the above

5 Given $(p+q)^2 = 4pq$ and T has coordinates $\left(apq, a(p+q)\right)$ then the locus of T is:

(A) $x^2 = 4ay$ (B) $x^2 = \dfrac{4y}{a}$ (C) $y^2 = 4ax$ (D) $y = 4ax^2$

6



The graph shown above represents the equation:

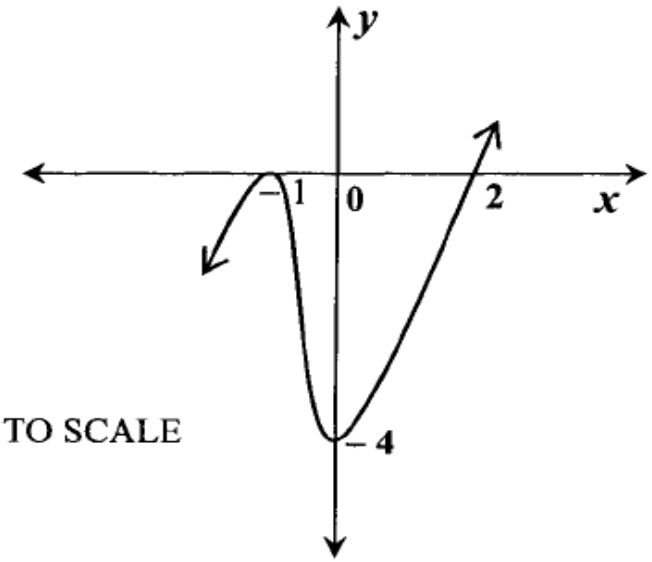
(A) $y = 2\sin x$

(B) $y = -2\cos x$

(C) $y = 2\sin\left(\dfrac{x}{2}\right) - 2$

(D) $y = -2\cos^2(2x)$

7 A cubic curve touches the x-axis at – 1 and intersects it at 2. The curve also intersects the y-axis at –4 as shown below. The equation of the curve is:



NOT TO SCALE

(A) $y = -2(x-1)^2(x+2)$ (B) $y = 2(1-x)^2(2-x)$

(C) $y = 2(x+1)^2(x-2)$ (D) $y = (x+1)^2(x-2)$

8 α, β and γ are the roots of the $x^3 - 4x^2 + 2x - 4 = 0$. The value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) -2

(D) $-\frac{3}{2}$

9 The function $y = e^{-kx}$ satisfies the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$, for some constant k. The possible values of k are:

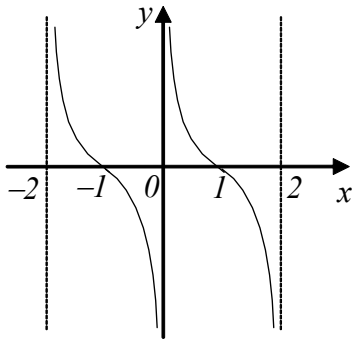
(A) $k = -1$ or 3

(B) $k = 1$ or 3

(C) $k = 1$ or -3

(D) $k = -1$ or -3

10 The diagram shows the graph of a function on the interval $-2 < x < 2$.



Which function does the graph represent?

(A) $y = -\tan \pi x$

(B) $y = \tan^{-1}\left(\frac{1}{x}\right)$

(C) $y = \cot\left(\frac{\pi}{2}x\right)$

(D) $y = \tan^{-1}\left(\frac{\pi x}{2}\right) - \cot^{-1}\left(\frac{\pi x}{2}\right)$.

End of Section I

Section II

60 marks
Attempt questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Find $\int \left(e^x + e^{-x}\right) dx$. **2**

(b) Find the coordinates of the point P which divides the interval joining $A(6, -3)$ to $B(-4,2)$ internally in the ratio 3:2. **2**

(c) Two roots of $2x^3 - x^2 + kx - 4 = 0$ are reciprocals of one another. Find the value of k. **2**

(d) The function $f(x) = x - \cos x$ has a zero near $x = 1$. **3**

Use one application of Newton's method to obtain another approximation to this zero. Give your answer correct to two decimal places.

(e) For the function $y = 3\cos^{-1}(2x)$:

(i) State the domain **1**

(ii) State the range **1**

(iii) Sketch the curve. **1**

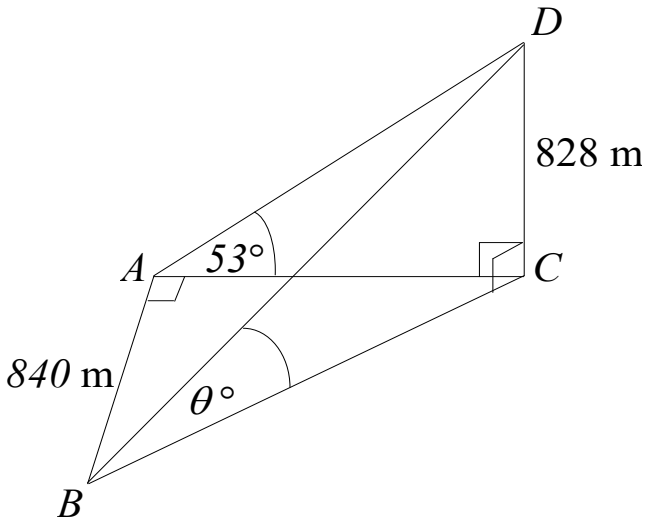
(f) Find the coordinates of the points on the line $y = 2x$ that are $\sqrt{2}$ units from the line $y = 4 - x$ **3**

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Use mathematical induction to prove that, for all counting numbers n, $2^{3n}-1$ is divisible by 7. **3**

(b) Use long division to divide $P(x)=x^3-x-3$ by $x-2$. Express your answer in the form $P(x)=(x-2)Q(x)+R$ where $Q(x)$ is the quotient and R is the remainder. **2**

(c) The world's tallest man-made structure is the Burj Khalifa tower building located in Dubai. It is approximately 828m. At a place A due west of it, the angle of elevation is $53°$. At a place B due south of A, the elevation is θ. The distance from A to B is 840 metres.

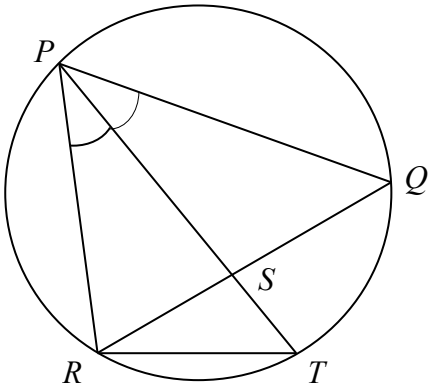


(i) Prove $\cot\theta=\sqrt{\dfrac{828^2\cot^2 53°+840^2}{828^2}}$ **3**

(ii) Find θ to the nearest degree. **1**

Question 12 continues on page 6

(d) In the diagram, the bisector of $\angle RPQ$ meets RQ in S and the circle again at T.



Copy the diagram into your writing booklet.

(i) Prove that the triangles PSQ and PRT are similar. **2**

(ii) Show that $PQ \times PR = PS \times PT$. **1**

(iii) Hence, prove that $PS^2 = PQ \times PR - RS \times SQ$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

Consider the function defined by $f(x) = \ln(x) + \frac{1}{x} - 1$.

(a) What is its domain? **1**

(b) What is the behaviour of the function as $x \to 0^+$? **1**

(c) What is the behaviour of the function as $x \to \infty$? **1**

(d) Show that $f'(x) = \frac{x-1}{x^2}$ and hence determine the coordinates of the stationary point. **3**

(e) Solve $\frac{x-1}{x^2} > 0$ and hence find for what values of x the function is: **2**

I. increasing II. decreasing

(f) Using your answers to (d) and (e), what is the range of the function? **1**

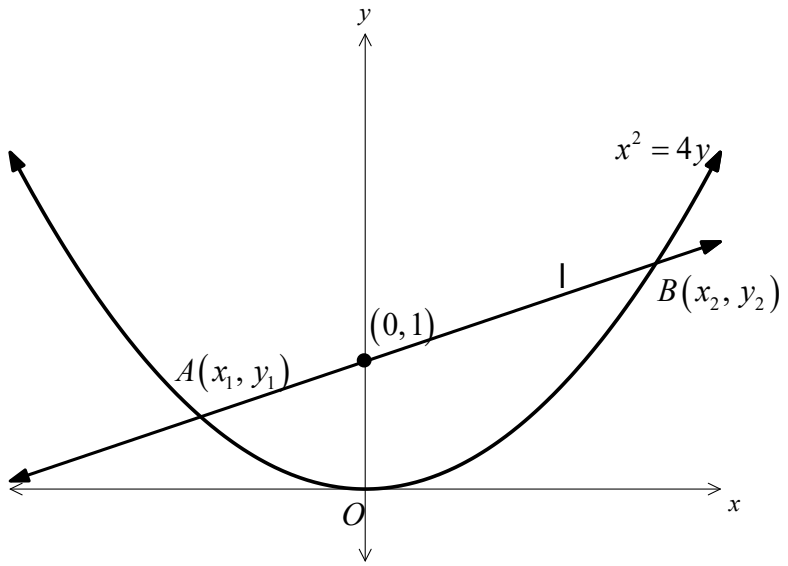
(g) Explain why for $x > 0,\ x \neq 1$: $\frac{x-1}{x} < \ln x$. **2**

(h) If $b > a > 0$ deduce the inequality $\frac{1}{b} < \frac{\ln b - \ln a}{b-a} < \frac{1}{a}$ **3**

(i) Using the result in (h), between which two fractions does $\ln(1.01)$ lie? **1**

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a)



A straight line l with gradient $m > 0$ passes through the point $(0,1)$.
The line l cuts the parabola $x^2 = 4y$ at two points $A(x_1, y_1)$ and $B(x_2, y_2)$ as shown above.

(i) Find the equation of l. **1**

(ii) Line l cuts the parabola at two distinct points. **2**
Show that x_1 and x_2 are the roots of the equation $x^2 - 4mx - 4 = 0$.

(iii) Show that $(x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_1x_2$.
Hence, or otherwise find $(x_2 - x_1)^2$ in terms of m. **2**

(iv) From (iii), show that $AB = 4(1 + m^2)$. **3**

C is a circle with AB as a diameter.

(v) Find in terms of m the coordinates of the centre of circle C. **2**

(vi) Find in terms of m, the distance from the centre of circle C to the line $y + 1 = 0$. **1**

(vii) Hence or otherwise, state the geometrical relationship between circle C and the line $y + 1 = 0$. Explain your answer. **1**

(b) Prove $\dfrac{1 + \cos\theta - \sin\theta}{1 + \cos\theta + \sin\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$ **3**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$