2013
HALF YEARLY
Examination

Number: _____

Teacher: _____

Year 12 Extension 1 Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Section I ~ Pages 1-4

- 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II ~ Pages 5-9

- 60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Teachers:

Mr Bradford (Examiner)
Mr Johansen
Mr Mulray
Mr Vuletich

Write your Board of Studies Student Number on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.

Number of Students in Course:

BLANK PAGE

Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 The expression $(1 - x^2)(1 + x^2 + x^4)$ is equal to:

(A) $1 - x^6$

(B) $1 - x^2 - x^4 - x^6$

(C) $1 + x^4 + x^6$

(D) $1 - x^2 + x^4 - x^6$

2 A is the point $(-2, -1)$. B is the point $(1, 5)$. The coordinates of the point Q, which divides AB externally in the ratio $5 : 3$ are:

(A) $\left(5\frac{1}{2}, 3\frac{1}{2}\right)$

(B) $\left(\frac{1}{8}, 2\frac{3}{4}\right)$

(C) $\left(5\frac{1}{2}, 14\right)$

(D) None of the above

3 What is the value of $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 2x}$?

(A) undefined

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{2}{\pi}$

4 If the solutions of $x^2 + bx + 36 = 0$ are integers, then the number of integer values b can have is:

(A) 7

(B) 8

(C) 9

(D) 10

5 If $f(x) = 10x$ and $f(g(x)) = -5x$, then $g(x)$ equals:

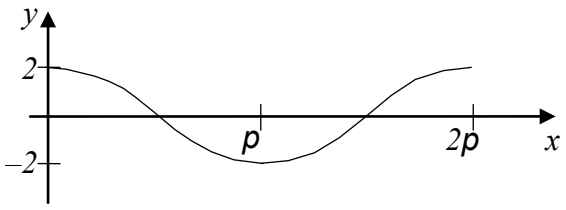
(A) $-\frac{1}{2}$

(B) $-\frac{x}{2}$

(C) $-\frac{x}{10}$

(D) $-2x$

6



The graph shown represents the equation:

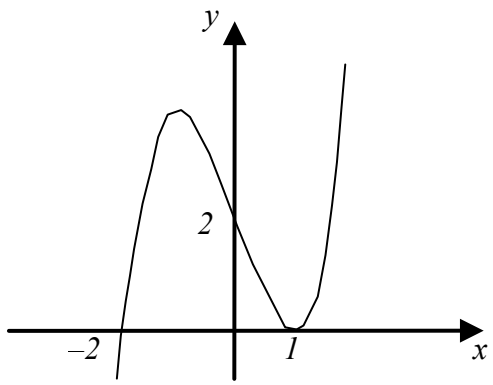
(A) $y = 2\sin x$

(B) $y = 2\cos x$

(C) $y = 2\sin\left(\frac{x}{2}\right)$

(D) $y = \frac{1}{2}\cos(2x)$

7



If in the diagram the graph represents a cubic, then the equation of the graph is:

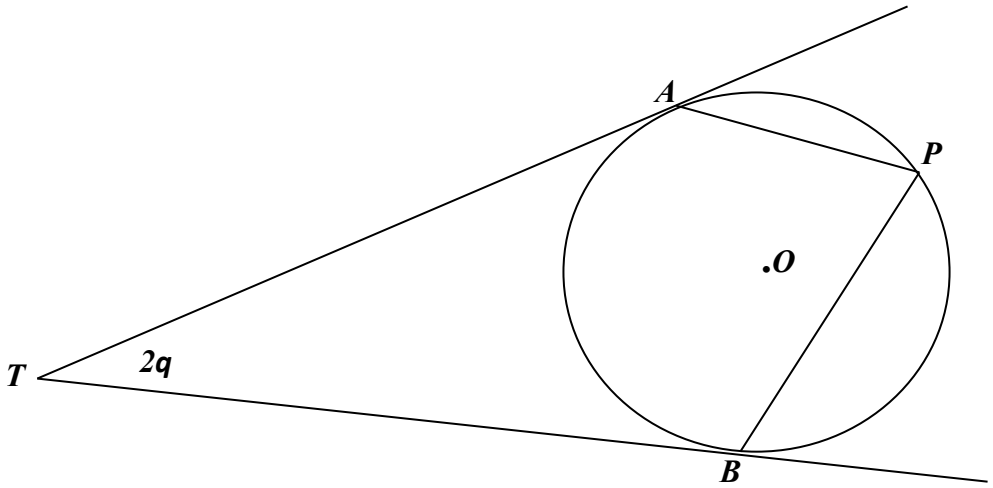
(A) $y = -(x-1)^2(x+2)$

(B) $y = (1-x)^2(2-x)$

(C) $y = (x+1)^2(x-2)$

(D) $y = (x-1)^2(x+2)$

8 Consider the diagram below.



The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T and $\angle ATB = 2\theta$. What is $\angle APB$ in terms of θ?

(A) θ

(B) $90° - \theta$

(C) $180° - 2\theta$

(D) $90° - \dfrac{\theta}{2}$

9 If $\tan A = \dfrac{2rs}{r^2-s^2}$, where A is acute and $r > s > 0$, then $\cos A$ is equal to:

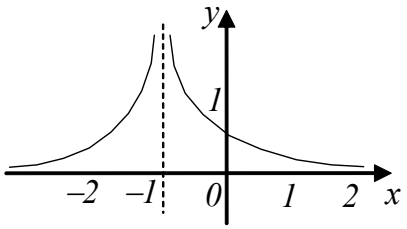
(A) $\dfrac{r^2-s^2}{r^2+s^2}$

(B) $\dfrac{\sqrt{r^2-s^2}}{2r}$

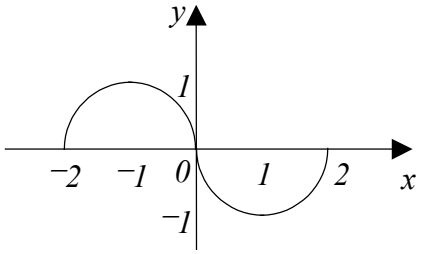
(C) $\dfrac{rs}{r^2+s^2}$

(D) $\dfrac{r}{r-s}$

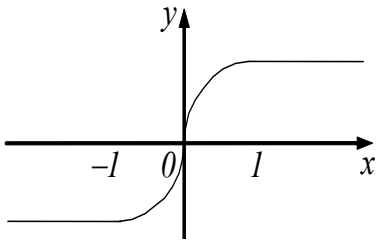
10



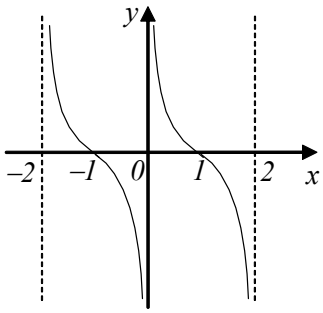
(I)



(II)



(III)



(IV)

The graphs of four functions are shown above. In which cases is the inverse relation also a function?

(A) None of the above

(B) (II) and (III) only

(C) (I) and (IV) only

(D) (I), (II), (III) and (IV)

End of Section I

Section II

60 marks
Attempt questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\int_0^2 \frac{x^2}{x^3+1}\,dx$. **2**

(b) If $xy = 7$, find the value of $\frac{2^{(x+y)^2}}{2^{(x-y)^2}}$. **2**

(c) Solve $\frac{1}{x-1} > 1$ **2**

(d) The straight lines $4x + 3y + p = 0$ and $5x - 12y + 34 = 0$ are equidistant from the point $(1, 0)$. Find the two possible values of p. **2**

(e) For a given function, its derivative is given by $\frac{dy}{dx} = 3x^2 - 2x + k$.
The curve corresponding to this function is stationary at the point $(2, -5)$. Find

(i) the value of k **1**

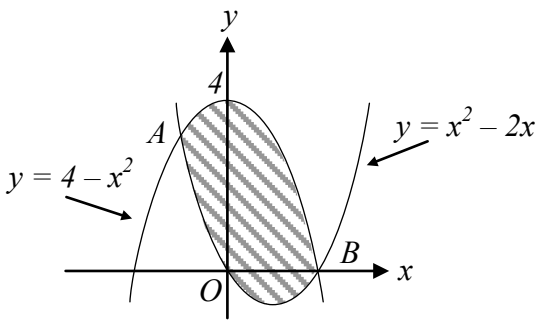
(ii) the equation of the curve. **2**

(f) (i) Expand and simplify $(x+y)^3$ **1**

(ii) If x is a positive real number and $(x + \frac{1}{x})^2 = 7$, calculate $x^3 + \frac{1}{x^3}$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a)



NOT TO SCALE

The curves $y = 4 - x^2$ and $y = x^2 - 2x$ intersect at the points A and B as shown in the diagram above.

(i) Find the x coordinates of the points A and B. **2**

(ii) Calculate the area of the shaded region. **2**

(b) Find the roots of the polynomial equation $9x^3 - 27x^2 + 11x + 7 = 0$, if the roots form an arithmetic sequence. **3**

(c) (i) Sketch on the same number plane the graphs of $y = \ln(1+x)$ and $y = \frac{1}{2}x$. **2**

(ii) Use your sketch to write down the smaller solution to $\ln(1+x) = \frac{1}{2}x$. **1**

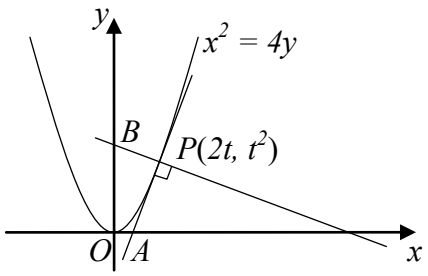
(iii) Between what two consecutive integers does the greater solution lie? **1**

(iv) Taking the midpoint of your answer in part (iii) as a first approximation, use one application of Newton's method to approximate the greater solution. Give your answer correct to 3 decimal places. **2**

(d) What is the coefficient x^{99} in the expansion of $(x-1)(x-2)(x-3)\ldots(x-100)$? **2**

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a)



The diagram shows the graph of the parabola $x^2 = 4y$. The tangent to the parabola at $P(2t, t^2)$, $t \neq 0$, cuts the x axis at A. The normal to the parabola at P cuts the y axis at B.

(i) Derive the equation of the tangent AP. **2**

(ii) Show that B has coordinates $(0, t^2 + 2)$. **2**

(iii) Let C be the midpoint of AB. The locus of C is a parabola. **3**
Find the cartesian equation of the locus and explain carefully why the vertex of the parabola is not part of the locus.

(b) Suppose $a^k - 2$ is even for *all* positive integers k and some *fixed odd* counting number a with $a > 1$.

(i) If as supposed $a^k - 2$ is even, explain why $a^{k+1} - 2$ must also be even. **2**

(ii) On the basis of this evidence, a Year 12 Extension 1 student assumed incorrectly that $2013^k - 2$ is even for *all* positive integers k. Use your knowledge of mathematical induction to explain the flaw in the student's reasoning and state the correct result concerning $a^k - 2$ when a is odd. **2**

(c) Let $f(x) = g(x) - \ln\left[g(x) + 1\right]$

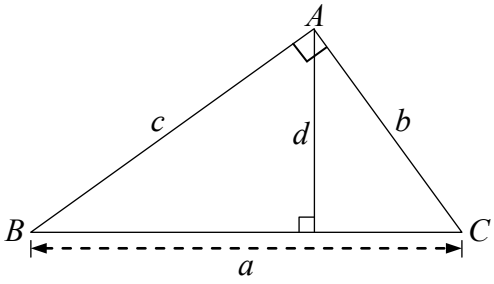
(i) Prove that $f'(x) = \dfrac{g(x).g'(x)}{g(x)+1}$. **2**

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\pi/2} \frac{\sin x \cos x}{\sin x + 1}\,dx$. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

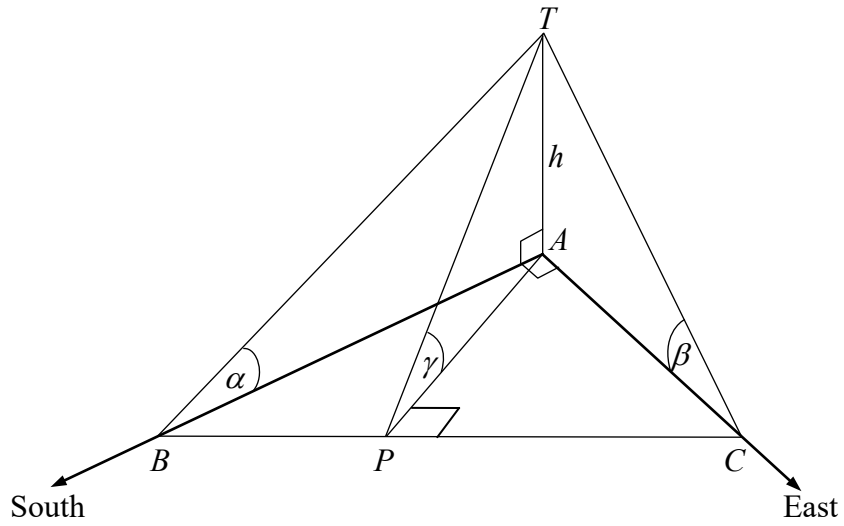
(a) By using long division, or otherwise, find the quotient and remainder when $x^{100} + x^2 + 1$ is divided by $x^2 - 1$. **3**

(b) (i)



The triangle ABC is right-angled at A and has sides with lengths a, b and c, as shown in the diagram above. The perpendicular distance from A to BC is d. By considering areas, or otherwise, show that $b^2c^2 = d^2(b^2 + c^2)$. **2**

(ii)



The points A, B and C lie on a horizontal surface. The point B is due south of A. The point C is due east of A. There is a vertical tower, AT, of height h at A. The point P lies on BC and is chosen so that AP is perpendicular to BC. Let α, β and γ denote the angles of elevation to the top of the tower from B, C and P respectively. Using the result in part (i), or otherwise, show that $\tan^2\gamma = \tan^2\alpha + \tan^2\beta$. **3**

Question 14 continues on page 9

Question 14 (continued)

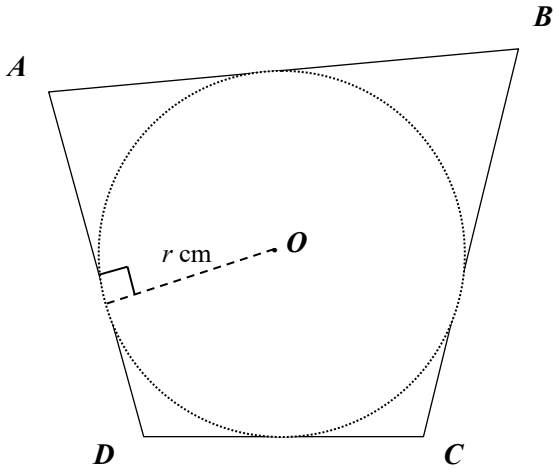
(c) (i) Write down an expression for the n^{th} term of the sequence 3, 5, 7, 9, ... **1**

(ii) Use the result in (i) to find the value of the product: **3**

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\left(1+\frac{9}{16}\right)\ldots\left(1+\frac{61}{900}\right)$$

Explain your reasoning carefully.

(d)



A quadrilateral contains as inscribed circle of radius r cm as shown in the diagram. **3**

Copy or trace the diagram into your writing booklet.

The ratio of the perimeter of the quadrilateral to that of the circle is *4 : 3*. Using your knowledge of circle geometry, find the ratio of the area of the quadrilateral to that of the circle.

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$