KNOX GRAMMAR SCHOOL

VIRILE AGITUR

TERM TWO EXAMINATION

I.M.

1993

MATHEMATICS

3 UNIT/ 4 UNIT

FORM 6

Time allowed: Three hours
(includes reading time)

INSTRUCTIONS

ALL questions should be attempted
ALL questions are of equal value.
ALL necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved calculators may be used.
Each question should be started on a new page.

The papers are to be handed in in the following manner:

Part A	**Questions 1-2**
Part B	**Questions 3-4**
Part C	**Questions 5**
Part D	**Questions 6**
Part E	**Questions 7**

Name: TIM SADLER **Class:** 3unit-

PART A

QUESTION 1 12 marks

(a) Factorise completely $a^4 - b^4 - a^2 - b^2$

(b) A and B are the points (-4,3) and (2,-1) respectively. Find the coordinates of the point Q which divides AB externally in the ratio 4:5.

(c) Using the expansion for $\tan(\alpha - \beta)$ find, with a rational denominator, the exact value of $\tan 15°$.

(d) Solve for x: $\frac{x^2 - 4x + 3}{x - 2} \geq 0$

e) Draw the graph of a function $y = f(x)$, $1 \leq x \leq 5$ such that $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$ for $1 \leq x \leq 5$.

QUESTION 2 12 marks **start a new page**

(a) Find the value of the expression, $\cos^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$ in terms of π in simplest form.

(b) Find

i) $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1 - 4x^2}}$

ii) $\int \cos\theta \sin^3\theta \, d\theta$ where $u = \sin\theta$

(c) If α, β, γ are the roots of the equation: $2x^3 - 3x^2 + 5x - m = 0$, **find:**

i) the value of $\alpha + \beta + \gamma$ and $\alpha^2 + \beta^2 + \gamma^2$

ii) the value of m if $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -2$

PART B

QUESTION 3 **12 marks** **Start a new page**

(a) Use mathematical induction to prove that $7^n - 1$ is always divisible by 6 for all positive integers n.

(b) When the polynomial $3x^3 - mx^2 - nx + 1$ is divided by $(x-2)$, the remainder is 15; $(x-1)$ is a factor of the polynomial. Find m and n.

(c) i) Show that $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$

ii) Hence evaluate: $\int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{1+\cos 2x}\,dx$

QUESTION 4 **12 marks** **Start a new page**

(a) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$

i) PQ passes through (0,4a). Show that pq = -4.

ii) Find the coordinates of M, the midpoint of PQ.

iii) Find the locus of M for all positions of P and Q.

(b) Show that the equation: $\sin x - 3\cos x = \tan\frac{x}{2}$ may be written as:

$t^3 - 3t^2 - t + 3 = 0$ where $t = \tan\frac{x}{2}$. Hence find all the solutions of the equation for $0° \le x \le 360°$, giving answers to the nearest minute where necessary.

PART C

QUESTION 5 **12 marks** **Start a new page**

(a) Find $\frac{d}{dx}\sin^{-1}(\cos 2x)$ leaving your answer in simplest form.

(b) Use the substitution $u = \sqrt{x}$ to evaluate $\int_1^3 \frac{dx}{(1+x)\sqrt{x}}$, giving your answer in exact form.

(c) i) State the domain and range of $y = \sin^{-1}(2x)$ and sketch the curve.

ii) Make x the subject if $y = \sin^{-1}(2x)$

iii) Find the volume generated when the area bounded by $y = \sin^{-1}(2x)$, the y-axis and $y = \frac{\pi}{2}$ is rotated about the y-axis.

PART D

QUESTION 6 **12 marks** **Start a new page**

(a) Find **all** the values of α for which $\cos^2\alpha + \frac{\sqrt{3}}{2}\sin 2\alpha = 0$

(b) A body is projected with speed 24.5 m/s from the top of a cliff, 58.8 m high, at an angle α to the horizontal, where $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$. Take the bottom of the cliff as the origin and take the acceleration due to gravity g as 9.8 m/s^2.

i) Show that $x = 14.7t$ and $y = 58.8 + 19.6t - 4.9t^2$.

ii) Find the range on the horizontal plane through the foot of the cliff.

iii) Find the speed of the body when it reaches this point.

PART E

QUESTION 7 **12 marks** **Start a new page**

(a) When x m from the origin on a straight line, the velocity, v m/s, of a particle is given by

$v = 1 + e^{-x}$

i) Find its acceleration when at the origin.

ii) Describe the velocity of the particle as it moves to the right of the origin.

(b) A particle moves on a straight line so that, after t seconds, it is x m from the origin and

$x = A\cos nt$, A and n are constant.

i) Prove $\ddot{x} = -n^2x$

ii) Prove that its velocity v m/s is given by $v^2 = n^2(A^2 - x^2)$.

iii) At x = 4, it has acceleration -1 m/s^2 and speed 1.5 m/s.

Find: (a) the period
(b) the amplitude
(c) the maximum speed of the motion.

END OF PAPER