Year 12 3 unit Mathematics
Answers to 1999 Half-Yearly Examination

Question 1

(a) $1\frac{1}{3}$

(b) 7.9

(c) $m = 2, n = 1$

(d) $\{x: x < 1\frac{1}{2}, x \geq 2\}$

Question 2

(b) $e^{3x}(3\cos x - \sin x)$

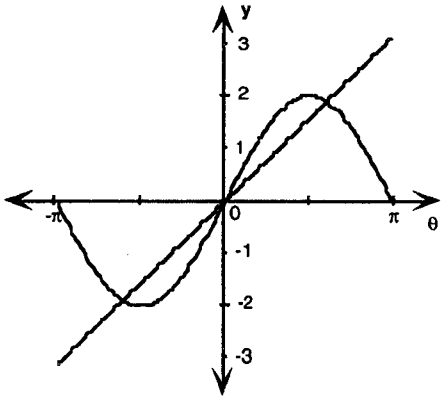
(c) 1.386

(d) 26°34'

(e) $\dfrac{\sqrt{2}-1}{2}$

Question 3

(a) (i)



(ii) 3 solutions

(c) (i) $L = 1$

(ii) $x > 10$

(d) (ii) 13 minutes

Question 4

(a) $\frac{9}{28}$

(b) 21 000 litres

(c) $4e^3$ ms^{-2}

(d) (iii) $d^2\left(\dfrac{4\pi - 3\sqrt{3}}{6}\right)$ cm^2

Question 5

(a) (i) $\frac{2197}{8000}$

(ii) $\frac{5803}{8000}$

(b) (i) $x = t^4 - 8t^3 + 10t^2 + 3$

(ii) Maximum turning point

(iii) 0.7 mg

(c) 2e

Question 6

(a) (i) $\dfrac{2x}{x^2-4}$

(ii) $\dfrac{2}{\ln 3(2x+5)}$

(b) (i) $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

(ii) $1 - 2\sin^2 x$

(iii) $A \equiv \left(\dfrac{\pi}{6}, 0\right)$ and $B \equiv \left(\dfrac{5\pi}{6}, 0\right)$

(iv) $\pi\left(2\pi - 3\sqrt{3}\right)$ cubic units

Question 7

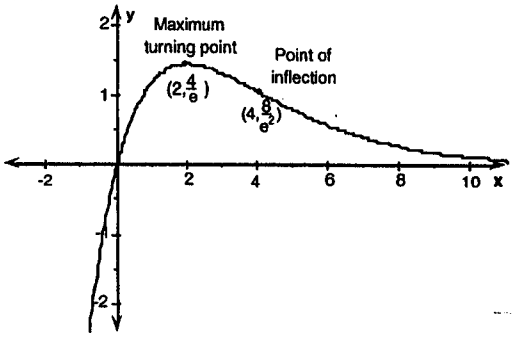
(b) (ii) $1 - \dfrac{2}{e}$

(c) (i) Maximum turning point at $\left(2, \dfrac{4}{e}\right)$

Point of inflection at $\left(4, \dfrac{8}{e^2}\right)$

(ii) 0

(iii)



KAMBALA CHURCH OF ENGLAND GIRLS' SCHOOL

MATHEMATICS

3 UNIT

YEAR 12 HALF-YEARLY EXAMINATION

APRIL 1999

Time Allowed: 2 hours
Reading Time: 5 minutes

INSTRUCTIONS

- Calculators and geometric instruments may be used.
- Each question is of equal value. Answer all questions on the writing paper provided.
- Start each question on a new page.
- Show all necessary working.
- Marks will be deducted for careless or badly arranged work.

Question 1 *(Start a new page)*

(a) Find $\lim_{x \to 0} \frac{4x}{\sin 3x}$.

(b) Given that $\log_a 2 = 2.8$ and $\log_a 3 = 4.1$, find $\log_a 6a$.

(c) The point P(10,-5) divides the interval joining A(6,1) and B(12,-8) in the ratio m : n. Find values for m and n.

(d) Solve for x: $\frac{1}{2x-3} \leq 1$

Question 2 *(Start a new page)*

(a) By completing the square, or otherwise, show that the expression $10 + 2x - x^2$ can never be greater than 11.

(b) Find $\frac{d}{dx}\left(e^{3x} \cos x\right)$.

(c) Given that $xy = 1$, evaluate $\int_1^4 y \, dx$ to 4 significant figures.

(d) Find the size of the acute angle between the graphs of $y = x$ and $y = x^3$ at the point of intersection when $x = 1$.

(e) Using the table of standard integrals, find the exact value of $\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \, dx$.

Question 3 *(Start a new page)*

(a) (i) On the same set of axes, sketch the graphs of $y = 2 \sin \theta$ and $y = \theta$ for $-\pi \leq \theta \leq \pi$.

(ii) Use your sketch to find the number of solutions of the equation $2 \sin \theta = \theta$ for $-\pi \leq \theta \leq \pi$.

(b) Prove that $24^n + 6^n = 2^n.3^n(4^n + 1)$.

(c) Given that $f(x) = \dfrac{x^3 + 1}{x^3}$:

(i) find L, such that $L = \lim\limits_{x \to \infty} f(x)$.

(ii) find values of x such that $f(x) - L < 0.001$.

(d) The rate of leakage of water out of a container is proportional to the amount of water in the container at any one time, i.e. $\dfrac{dV}{dt} = -kV$.

(i) Show that $V = V_0e^{-kt}$ is a solution of this equation.

(ii) If the container is 60% empty after 5 minutes, find how long it will take for the container to be 90% empty. Give your answer to the nearest minute.

Question 4 *(Start a new page)*

(a) A class of 28 students puts on a concert with all class members performing. If 15 class members dance and 19 sing in the performance, find the probability that any one student chosen at random from the class will **only** dance.

(b) The volume rate of flow of water (in litres/hour) into a dam is given by:

$R = 500 + 20t$

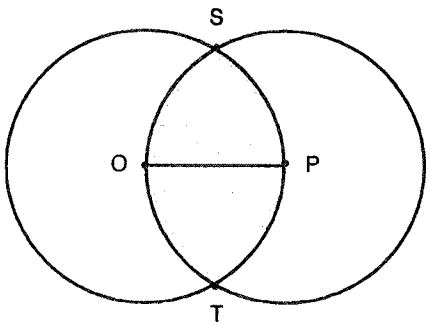
If there is 15 000 L of water initially in the dam, how much water will there be in the dam after 10 hours?

(c) The displacement (in metres) of a particle at any time, t seconds, is given by:

$x = e^{2t} + 1$

Find the exact acceleration after 1 second.

(d) The points O and P are d cm apart. A circle centre O is drawn to pass through P and another circle centre P is drawn to pass through O. The two circles meet at S and T, as shown in the diagram below.



(i) Show that ΔSOP is equilateral.

(ii) Show that $\angle SOT = \frac{2\pi}{3}$.

(iii) Hence find the area of the shaded region in terms of d.

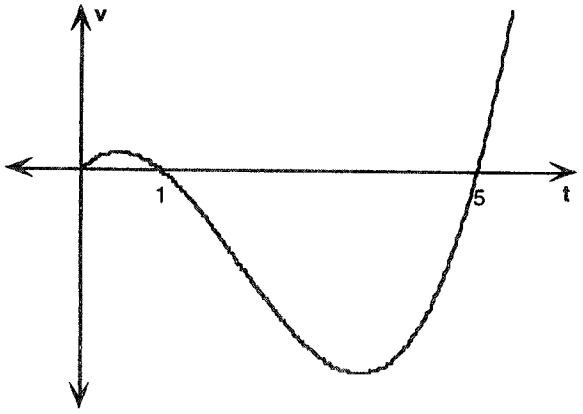
Question 5 *(Start a new page)*

(a) A certain breed of cat has a 35% chance of producing a white kitten. If a cat has 3 kittens, find the probability that she will produce:

(i) no white kittens.

(ii) at least one white kitten.

(b) A pen moves along the x-axis, ruling a straight line. The graph below represents the **velocity** of the tip of the pen as a function of time.



The velocity, in cm/s, is given by $v = 4t^3 - 24t^2 + 20t$. The tip of the pen is initially 3 cm to the right of the origin.

(i) Find an expression for x as a function of t.

(ii) What feature will the graph of x (as a function of t) have at t = 1?

(iii) The pen uses 0.05 mg of ink per cm travelled. How much ink is used during the time interval between t = 0 and t = 2?

(c) Find the exact gradient of the tangent to the curve $y = e^{x + \ln x}$ at the point where x = 1.

Question 6 *(Start a new page)*

(a) Differentiate the following functions with respect to x:

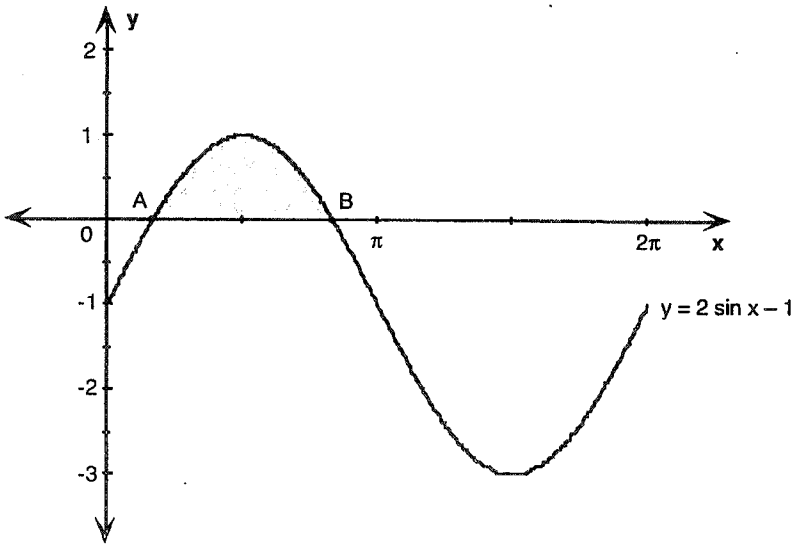
(i) $f(x) = \ln (x^2 - 4)$

(ii) $f(x) = \log_3 (2x + 5)$

(b) (i) Write down the expansion of $\cos (\alpha + \beta)$.

(ii) Hence deduce an expression for $\cos 2x$ in terms of $\sin x$.

(iii)



The diagram above shows the graph of $y = 2 \sin x - 1$ for $0 \leq x \leq 2\pi$. A and B are both points on the x-axis. Find the co-ordinates of A and B.

(iv) Using your answer to (ii), or otherwise, find the volume formed when the shaded area is rotated around the x-axis.

Question 7 *(Start a new page)*

(a) Use the Principle of Mathematical Induction to prove that $5^n + 2(11^n)$ is a multiple of 3 for all positive integers n.

(b) (i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1 - \ln x}{x^2}$.

(ii) Hence evaluate $\int_1^e \frac{1}{x^2} \ln x \ dx$.

(c) Consider the function $y = 2xe^{-0.5x}$

(i) Find any stationary points and determine their nature.

(ii) Find the limiting value of y as $x \to \infty$.

(iii) Sketch the curve $y = f(x)$ showing all definitive features.

END OF EXAMINATION