KAMBALA CHURCH OF ENGLAND GIRLS' SCHOOL

HALF YEARLY EXAMINATION

MATHEMATICS

3 UNIT

YEAR 12

MAY 1997

Time allowed: 2 hours
(plus 5 minutes reading time)

INSTRUCTIONS:

1. All questions may be attempted.
2. All necessary working must be shown.
3. Marks may not be awarded for untidy or badly arranged work.
4. Approved calculators may be used.
5. Begin each question in a new booklet.
6. Each question is of equal value.

Question 1

a) Solve $\frac{2}{x-3} \geq 4$

b) Find the co-ordinates of the point P which divides QR externally in the ratio 3 : 2, given that Q is the point (-5, 6) and R is the point (-2, 3).

c) Find $\lim_{x \to 0} \frac{\sin 5x}{2x}$

d) If θ and α are angles between 0^0 and 90^0 and $\sin\theta = \frac{3}{5}$ and $\tan\alpha = \frac{7}{24}$ find $\cos(\theta + \alpha)$

e) Differentiate the following with respect to x:

i) $e^{\sec x}$

ii) $\tan^{-1}(3x)$

Question 2 (start a new page)

a) Find the size, to the nearest degree, of the acute angle between the lines $4x - y + 5 = 0$ and $6x + 3y = 7$

b) Using the substitution u= x + 4, find $\int 5x\sqrt{x+4}dx$

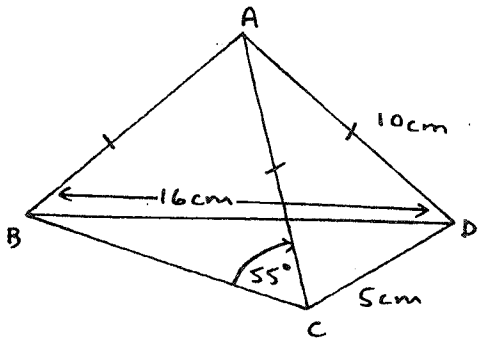
c) Find the equation of the tangent to the curve $y = e^{-x}\cos 2x$

at the point $(\frac{\pi}{4}, 0)$.

d) Find the primitive function of $\frac{2x}{x^2+1}$

Question 3 (start a new page)

a)



ABCD is a triangular pyramid with AB=AC=AD=10cm, BD=16cm, CD=5cm and $\angle BCA = 55^0$

i) Calculate the length of BC, to 1 decimal place

ii) Find $\angle BCD$, to the nearest minute.

b) Use the principle of mathematical induction to prove that $5^n + 3$ is divisible by 4 for all positive integers, n.

c) Find the general solution to the following equation

$\cos 2x = \cos x$

Question 4 (start a new page)

a) The velocity of a particle is given by $v = 4\cos 2t \ \ ms^{-1}$. If the particle is 3m to the right of the origin after π sec find:

i) the displacement as a function of time

ii) the exact acceleration after $\frac{\pi}{6}$ sec

b) i) Differentiate $y = \sin^{-1} x + x\sqrt{1-x^2}$

ii) Hence, or otherwise, evaluate $\int_0^{\frac{1}{2}} \frac{1-x^2}{\sqrt{1-x^2}}dx$

c) Show that

$$\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$$

Question 5 (start a new page)

a) Find the exact value of $\int_0^2 \frac{x}{\sqrt{1+x^2}}dx$ using the substitution $u = 1+x^2$

b) i) State the domain and range of the function

$$f(x) = 4\sin^{-1}\left(\frac{2x-1}{3}\right).$$

ii) Hence draw a neat sketch of this function for its given domain.

c) i) Write down an expression for $\cos 2x$ in terms of $\cos^2 x$.

ii) Use this expression to evaluate $\int_0^{\frac{\pi}{32}} \cos^2 8x dx$

Question 6 (start a new page)

a) The volume of an expanding balloon is increasing at a constant rate of $10cm^3s^{-1}$.

(Hint: volume of a sphere is $\frac{4}{3}\pi r^3$, surface area of a sphere $= 4\pi r^2$)

i) Find $\frac{dr}{dt}$ in terms of r.

ii) Determine the rate of increase of the surface area, S, of the balloon when its radius is 8cm.

b) Find the exact value of $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx$

c) i) Express $3\cos x - \sqrt{3}\sin x$ in the form $A\cos(x+\alpha)$.

ii) Sketch the function $y = 3\cos x - \sqrt{3}\sin x$ for $0 \le x \le 2\pi$.

iii) What is the maximum value of $3\cos x - \sqrt{3}\sin x$, and what is the smallest positive value of x for which it occurs?

iv) Solve $3\cos x - \sqrt{3}\sin x = \sqrt{6}$ for $0 \le x \le 2\pi$.

Question 7 (start a new page)

a) i) Sketch the function $y = \ln(x-1)$. Use at least half a page.

ii) Find the co-ordinates of the point P on the curve where $x = e+1$. Mark P on your sketch.

iii) Q is the foot of the perpendicular from P to the y axis.
R is the foot of the perpendicular from P to the x axis.
M is the point where the curve cuts the x axis.

Find the co-ordinates of Q, R and M and mark them on your sketch.

iv) Find the area bounded by the curve PM and the lines MR and PR. Shade this area on your sketch.

b) Consider the function $f(x) = \tan x + 3\cot x$ (x is in radians).

i) Show that there is only one stationary point in the interval $0 < x < \frac{\pi}{2}$

ii) Determine the nature of this stationary point.

END OF PAPER

Year 12 3 unit Mathematics
Answers to 1997 Half-Yearly Examination

Question 1

(a) $3 < x \leq 3\frac{1}{2}$

(b) (4,-3)

(c) $2\frac{1}{2}$

(d) $\frac{3}{5}$

(e) (i) $\sec x \tan x \, e^{\sec x}$

(ii) $\dfrac{3}{1+9x^2}$

Question 2

(a) 41°

(b) $2(x+4)^{\frac{5}{2}} - \frac{40}{3}(x+4)^{\frac{3}{2}} + C$

(c) $y = -2e^{-\frac{\pi}{4}}x + \frac{\pi}{2}e^{-\frac{\pi}{4}}$

(d) $y = \ln(x^2 + 1) + C$

Question 3

(a) (i) 11.4 cm

(ii) 150°3'

(c) $x = \dfrac{2n\pi}{3}$ or $2n\pi$

Question 4

(a) (i) $x = 2\sin 2t + 3$

(ii) $-4\sqrt{3}$ ms^{-2}

(b) (i) $\dfrac{2(1-x^2)}{\sqrt{1-x^2}}$

(ii) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{8}$

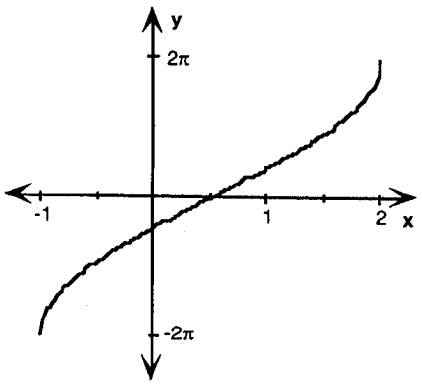
Question 5

(a) $\sqrt{5} - 1$

(b) (i) Domain: $\{x: -1 < x < 2\}$

Range: $\{y: -2\pi < y < 2\pi\}$

(ii)



(c) (i) $\cos 2x = 2\cos^2 x - 1$

(ii) $\dfrac{\pi + 2\sqrt{2}}{64}$

Question 6

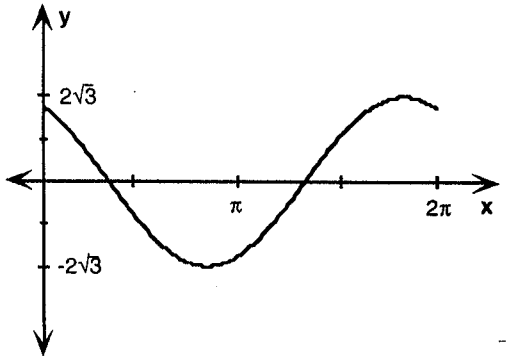
(a) (i) $\dfrac{5}{2\pi r^2}$

(ii) 2.5 cm^{2}s^{-1}

(b) $\dfrac{\pi}{6}$

(c) (i) $2\sqrt{3}\cos\left(x + \dfrac{\pi}{6}\right)$

(ii)



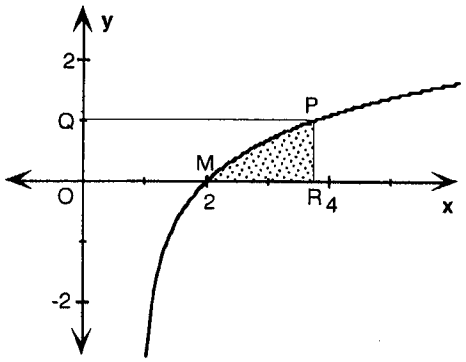
(iii) Maximum value = $2\sqrt{3}$ and occurs when $x = \dfrac{11\pi}{6}$

(iv) $x = \dfrac{\pi}{12}$ or $\dfrac{19\pi}{12}$

Year 12 3 unit Mathematics
Answers to 1997 Half-Yearly Examination

Question 7

(a) (i)



(ii) $P \equiv (e + 1, 1)$

(iii) $Q \equiv (0,1)$

$R \equiv (e + 1,0)$

$M \equiv (2,0)$

(iv) 1 square unit

(b) (i) $x = \frac{\pi}{3}$

(ii) Minimum turning point at $(\frac{\pi}{3}, 2\sqrt{3})$