257

Barker College

2006
YEAR 12 EXAMINATION TERM 1

Mathematics Extension 1

Staff Involved:

PM WEDNESDAY 12 APRIL

- **LJP***
- **GIC***
- **BTP**
- **JM**
- **CFR**

75 copies

General Instructions

- **Reading time – 5 minutes**
- **Working time – 2 hours**
- **Write using blue or black pen**
- **Make sure your Barker Student Number is on ALL pages**
- **Board-approved calculators may be used**
- **A table of standard integrals is provided on page 9**
- **ALL necessary working should be shown in every question**
- **Marks may be deducted for careless or badly arranged working**

Total marks – 84

- **Attempt Questions 1 – 7**
- **All questions are of equal value**

Total marks – 84
Attempt Questions 1 – 7
ALL questions are of equal value
Answer each question on a SEPARATE sheet of paper

Marks

Question 1 (12 marks) **[START A NEW PAGE]**

(a) L(4, –1) and P(–3, 9) are points on the number plane.

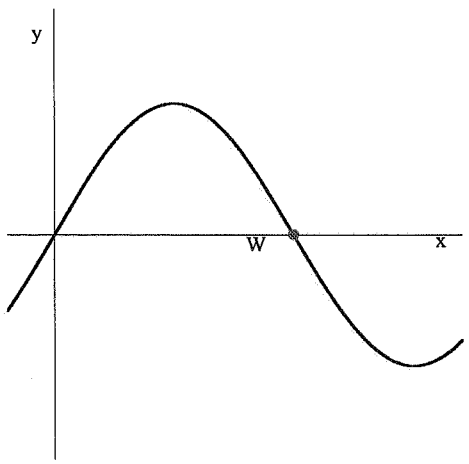
Find the coordinates of the point J, which divides the interval LP internally in the ratio 4 : 3. 3

(b) Find the obtuse angle between the lines $y = 2x - 3$ and $2x + 3y - 1 = 0$, to the nearest degree. 3

(c) Evaluate: $\lim_{x \to 0} \frac{3x}{\tan 2x}$ 2

(d) A part of the graph of $y = \sin\sqrt{3}x$ is shown.

Find the coordinates of w, in radians. 1



(e) Solve: $\frac{2}{x - 4} \geq 2$ 3

Question 2 (12 marks) **[START A NEW PAGE]**

(a) Using the table of standard integrals evaluate $\int_0^1 \frac{1}{\sqrt{x^2 + 4}}\, dx$ 2

(b) Sketch the graph of $y = 2\cos^{-1} 3x$, clearly showing domain, range and intercepts. 3

(c) Evaluate exactly:

(i) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ 1

(ii) $\sin\left(\sin^{-1}\frac{3}{8}\right)$ 1

(iii) $\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$ 2

(iv) $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{4}{3}\right)$ 3

Marks

Question 3 (12 marks) **[START A NEW PAGE]**

(a) Differentiate:

(i) $f(x) = \sin^{-1}(e^x)$ 2

(ii) $y = \cos^{-1}(\sin 3x)$ 2

(b) (i) Evaluate $\int_0^2 \frac{dx}{\sqrt{16 - x^2}}$ 2

(ii) Differentiate $x\sin^{-1}x + \sqrt{1 - x^2}$ and hence find $\int \sin^{-1}x\,dx$ 3

(c) The function $f(x) = e^{2-x} + 3$ has an inverse function $f^{-1}(x)$.

Find $f^{-1}(x)$ and state its domain and range. 3

Marks

Question 4 (12 marks) **[START A NEW PAGE]**

(a) Prove by Mathematical Induction that

$$1\times2 + 2\times3 + 3\times4 + \ldots + (n-1)n = \frac{1}{3}n(n-1)(n+1) \quad \text{for } n \geq 2$$ 4

(b) A sphere is expanding in such a way that its surface area is increasing by 16 cm^2/s. At what rate is its volume increasing when $r = 3$ cm? 4

(c) A cup of hot chocolate cools according to the equation $\frac{dT}{dt} = k(T - T_o)$, where T_o is the room temperature and T is the temperature in degrees after t minutes.

(i) Show that $T = T_o + Ae^{kt}$ is a solution of the equation above. 1

(ii) The room temperature is 20^o. If the drink cools from 85^o to 65^o in ten minutes, how long will it take to reach 40^o? 3

Question 5 (12 marks) **[START A NEW PAGE]**

(a) A particle, initially 1 metre to the right of the origin, O, moves according to the equation $v = t^2 - 6t + 8$

Find when and where the particle is first at rest. 3

(b) The velocity, v in cm/s, of a particle in relation to its displacement, x, is given by the equation $v = 2\sqrt{2(x - 3)}$

(i) Find the equation relating displacement, x, to time, t, if the particle is 5 cm to the right of the origin after 1 second. 3

(ii) When is the velocity 12 cm/s? 2

(c) A body is moving along a straight line according to the equation $\ddot{x} = -9x$

Initially, the body is 4 cm to the right of the origin and the velocity is 12 cm/s.

(i) Find the equation of the velocity, v, in terms of displacement. 2

(ii) What is the amplitude? 1

(iii) What is the maximum speed? 1

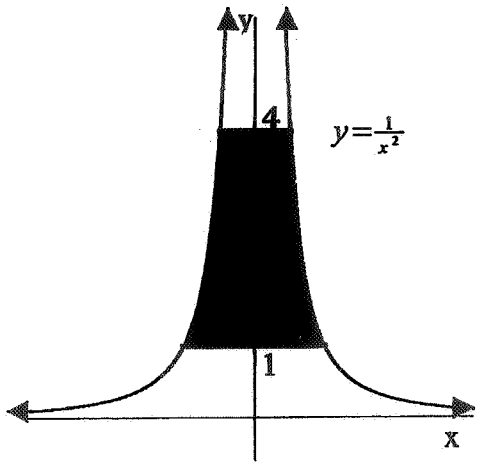
Marks

Question 6 (12 marks) **[START A NEW PAGE]**

(a) Find the exact value of $\sin 75^o$ 2

(b) Find the general solution of $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ 3

(c) The graph of $y = \frac{1}{x^2}$ is shown with the area shaded between the lines $y = 1$ and $y = 4$. Calculate the exact area. 4



(d) The graphs of $y = e^x$ and $y = \ln x$ have a common normal.

Find the equation of this normal.

[Hint: sketch the graphs and axis of symmetry]. 3

Question 7 (12 marks) **[START A NEW PAGE]**

(a) The area between the x-axis, the curve $y = \dfrac{1}{\sqrt{4 - x}}$, the y-axis and the line $x = 3$ is rotated about the x-axis.

Find the exact volume of the solid of revolution. 3

(b) Consider the function $f(x) = \dfrac{x^2 - 3}{e^x}$

(i) Locate all intercepts. 3

(ii) Find and determine the nature of all stationary points. 3

(iii) As $x \to \infty$, what value does $f(x)$ approach? 1

(iv) Hence, sketch the graph of $y = f(x)$, showing all features found. 2

End of Paper