SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS
Extension 1

Year 12

Assessment Task 2

March 2002

TOPICS : Exponential and Logarithmic Functions
The Trigonometric Function
Trigonometric Functions II

Time allowed- 75 minutes

DIRECTIONS TO CANDIDATES

- Attempt All questions
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Question 1 (12 marks)

(a) Differentiate with respect to x (i) $\log_e (3x+1)$

(ii) $x\sin x$

(iii) $\log_e \cos x$

(iv) $x\log_e x + \sin^2 x$

(b) Find the equation of the normal to the curve $y = \log_e x$ at the point where $x=e^2$

Question 2 (12 marks)

(a) Solve $e^{3t} = 5$ (correct to two decimal places)

(b) Find a primitive for (i) $\sqrt{x} - e^{-x}$

(ii) $\sin x + \sec^2 x$

(c) Sketch $y = \sin x$ and $y = \frac{2}{\pi}x$ with the same axes for $0 \le x \le \pi$.

Find the area enclosed between the two curves on this sketch.

Question 3 (12 marks)

(a) Evaluate (i) $\int_0^{\frac{\pi}{2}} \sin 2x \ dx$

(ii) $\int_3^4 \frac{x-1}{x^2-2x} dx$

(b) Write down the formula for the expansion of $\cos(A+B)$ and deduce that $2\sin^2\theta = 1 - \cos 2\theta$

(c) For the curve $y = f(x)$ it is given that $f'(x) = \sin^2 x$ and the curve passes through the point $(\frac{\pi}{4}, \frac{\pi}{8})$.

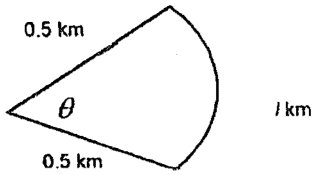
Find the equation of the curve.

Question 4 (12 marks)

(a) Find the acute angle between the lines $x - 2y = 0$ and $3x - y - 15 = 0$.

(b)

Figure not to scale



A car travels at 40 km/h on a circular curve whose radius is 0.5 km.

(i) Find the distance, *l* km, that the car travels in one minute.
(ii) Calculate the size of the angle θ through which the car turns in one minute. Give your answer to the nearest degree.

(c) Express $\sqrt{3}\sin x + \cos x$ in the form $C\sin(x + \alpha)$ where C is positive. (giving numerical values for C and α)

Question 5 (12 marks)

(a) Differentiate xe^x and hence, evaluate $\int_0^1 xe^x \, dx$.

(b) Calculate the volume of the solid generated when the region in the first quadrant bounded by the curve $y = e^x$, the line $x = \log_e 2$ and the co-ordinate axes,is rotated about the x-axis.

(c) If $0 \le \theta \le \frac{\pi}{2}$, prove that $\tan\theta = \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$

Hence, show that the exact value of $\tan\frac{\pi}{8}$ is $\sqrt{2} - 1$

Question 6 (15 marks)

(a) Solve $3t^2 + 4t - 4 = 0$ for $0 \le x \le 2\pi$ where $t = \tan\frac{x}{2}$.

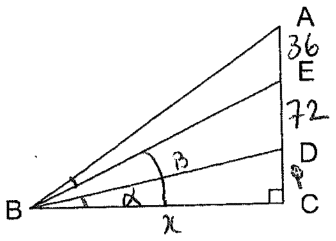
Give answer in radians, correct to two decimal places.

(b) Find the smallest positive value of θ if $\cos\theta - \sin 2\theta = 0$.

(c) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

and hence solve the equation $\cos 3\theta - \cos\theta = 0, \quad 0^\circ \le \theta \le 360^\circ$.

(d) The diagram shows a tall building AC. E and D are windows.
From a point B on horizontal ground it is found that
angle ABE= angle DBC. If DC = 9m, DE = 72m and AE = 36m.
<CBD = α, <EBC = β and BC = x metres.



(i) Show that $\dfrac{117}{x} = \dfrac{\frac{81}{x} + \frac{9}{x}}{1 - \frac{81}{x}\cdot\frac{9}{x}}$

(ii) Hence, find the value of x correct to two significant figures.

THE END