Name: _____

Teacher: _____

Class: _____

FORT STREET HIGH SCHOOL

2012

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 2

Mathematics Extension 1

TIME ALLOWED: 1½ HOURS

Outcomes Assessed	Questions	Marks
Applies the appropriate techniques from the study of series to solve problems		
Determines integrals by reduction to a standard form through a given substitution		
Manipulates algebraic expressions to solve problems involving inverse functions		

Question	1	2	3	4	5	Total	%
Marks	/15	/15	/15	/16	/16	/77	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each new question is to be started in a new booklet**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question One: (15 marks) **marks**

(a) For the sequence 9, 7, 5, …

i) Show that the sequence is arithmetic. **[1]**

ii) Find T_{13}. **[1]**

iii) Show that if the sum of n terms of the series is 21, then n may have two possible values. Find these values. **[3]**

iv) Explain the reasons for the existence of two values for n. **[1]**

(b) A frog begins to jump from one end of a log to the other end. The log is 2 metres long. Each jump carries the frog one quarter of the remaining distance to the other end. After 8 jumps, how far is the frog from the other end (to 4 dp)? **[2]**

(c) The series $\frac{1}{3}-\frac{1}{6}+\frac{1}{12}-...$ is geometric. Find the sum of the first 10 terms, giving you answer as a rational number in its lowest terms. **[3]**

(d) The sum of three consecutive terms in an Arithmetic Series is 21 and the sum of their squares is 155. Find the terms. **[4]**

Question Two: (15 marks) **marks**

(a) Using the substitution $u = 2 + x$, find $\int x(2+x)^4 dx$. **[3]**

(b) Evaluate $\int_0^{\frac{\pi}{6}} \cos^2 x\, dx$. **[3]**

(c) Use the substitution $u = x^4 + 4x + 1$ to find the integral (to 2 decimal places):

$$\int_0^1 \left(x^3+1\right) \sqrt[4]{x^4+4x+1}\ dx$$ **[4]**

(d) Find $\int \frac{1}{\sqrt{9-4x^2}} dx$ using the substitution $x = \frac{3}{2}\sin\theta$. **[5]**

Question Three: (15 marks) **marks**

(a) Given $f(x) = \sqrt{2-3x}$,

i) Draw a neat sketch of $f(x)$, stating its domain and range. **[2]**

ii) Find the inverse function $f^{-1}(x)$, and draw a neat sketch of it on the same graph as $f(x)$, stating its domain and range. **[4]**

(b) Evaluate $\tan\left(\tan^{-1}x - \tan^{-1}\left(\frac{1}{x}\right)\right)$. **[3]**

(c) Find the derivative of $x\ln\left(\cos^{-1}(x)\right)$. **[3]**

(d) Evaluate $\int_{-3}^{0} \frac{1}{x^2+9}dx$ **[3]**

Question Four: (16 marks) **marks**

(a) Kevin takes out a $280,000 home loan. Interest is charged at 9% p.a. and he agrees to repay the loan in monthly instalments over 25 years. How much is each monthly instalment (to the nearest $)? **[4]**

(b) Use the Principle of Mathematical Induction to show that $8^n \geq 5^n + 3^n$, for all integer n. **[4]**

(c) Without using a calculator, show that $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right) + \cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = \frac{\pi}{4}$. **[4]**

(d) Can there be an infinite geometric series with a limiting sum of $\frac{5}{8}$ and a first term of 2? Justify your answer (show all working and reasoning). **[4]**

Question Five: (16 marks) **marks**

(a) Find the equation of the curve whose gradient at any point x is $\frac{-1}{\sqrt{3-4x^2}}$ and which contains the point $\left(\frac{-3}{4},\frac{\pi}{3}\right)$. (Use the substitution $x=\frac{\sqrt{3}}{2}\cos\theta$). **[4]**

(b) Given $f(x)=\ln(\sec x+\tan x)$:

i) Show that $f'(x)=\sec x$ **[1]**

ii) Hence, show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sec x\,dx=\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$ **[3]**

(c) Evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}}\sin^2(x)\cos^2(x)\,dx$ **[4]**

(d) Prove by the Principle of Mathematical Induction that
$1\times 2^0+2\times 2^1+3\times 2^2+\ldots+n\times 2^{n-1}=1+(n-1)2^n$ for all integers $n\geq 1$. **[4]**