SYDNEY BOYS HIGH SCHOOL

2019

YEAR 11
TERM 4
HSC ASSESSMENT
TASK #1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 11–13, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 51

Section I – 10 marks (pages 2–6)
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 41 marks (pages 7–19)
- Attempt Questions 11–13
- Allow about 1 hour and 15 minutes for this section

Examiner:

External Examiner

NESA Number									

Name: _____ **Class: 11Ma** ___

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which expression is equivalent to $(\sin 2x + \cos 2x)^2$?

A. 1 B. $1+\sin 2x$ C. $1+\frac{1}{2}\sin 4x$ D. $1+\sin 4x$

2 What values of x satisfy $(x+1)(x-2)(x-3)<0$?

A. $(-1,2)\cup(3,\infty)$ B. $(-\infty,-1)\cup(2,3)$

C. $(-1,\infty)\cup(2,3)$ D. $(-1,3)$

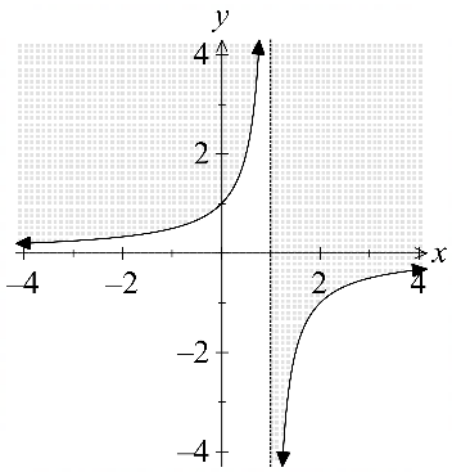
3 Which of the following shows the expansion of $\cos\left(2x+\frac{\pi}{6}\right)$?

A. $\frac{\sqrt{3}\cos 2x}{2}+\frac{\sin 2x}{2}$ B. $\frac{\sqrt{3}\cos 2x}{2}-\frac{\sin 2x}{2}$

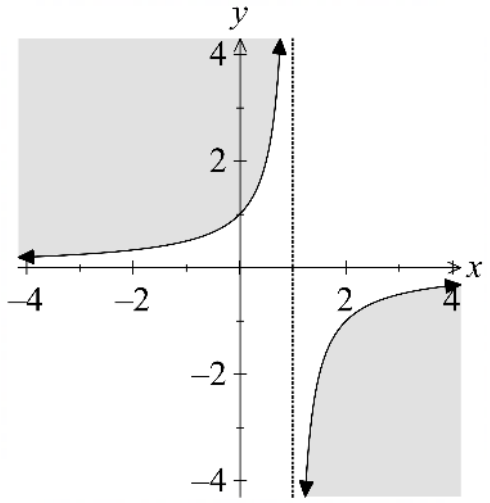
C. $\frac{\cos 2x}{2}+\frac{\sqrt{3}\sin 2x}{2}$ D. $\frac{\cos 2x}{2}-\frac{\sqrt{3}\sin 2x}{2}$

4 Which of the diagrams below shows the region defined by $y \geq -\dfrac{1}{x-1}$?

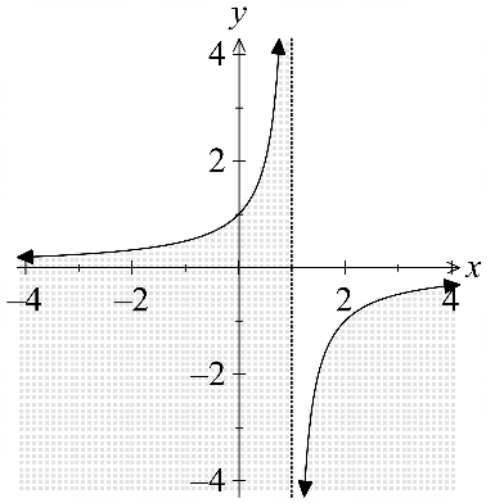
A.



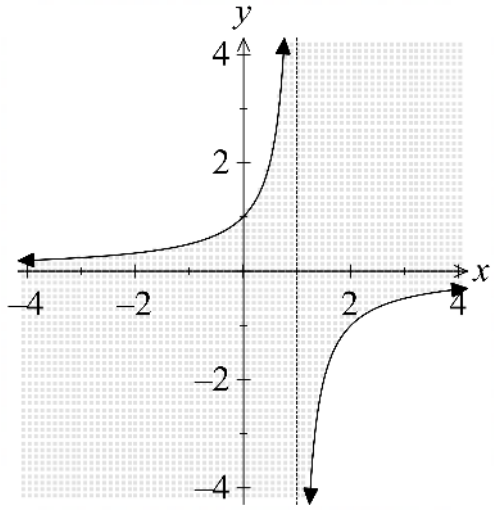
B.



C.



D.



5 Which of the following is equivalent to the expression $\sin\theta - \cos\theta$?

A. $2\sin\left(\theta - \dfrac{\pi}{4}\right)$

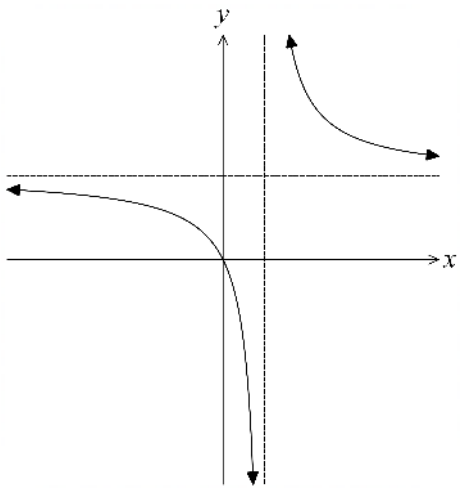
B. $2\sin\left(\theta + \dfrac{\pi}{4}\right)$

C. $\sqrt{2}\sin\left(\theta - \dfrac{\pi}{4}\right)$

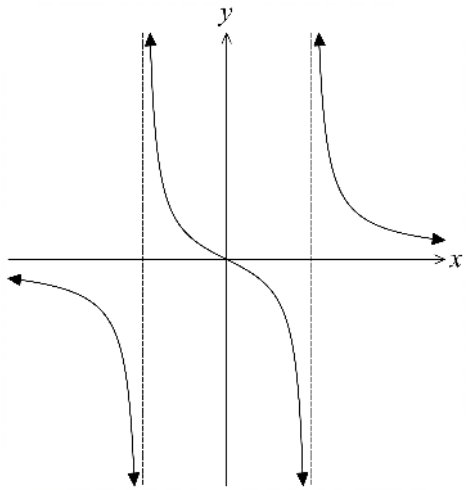
D. $\sqrt{2}\sin\left(\theta + \dfrac{\pi}{4}\right)$

6 There are four graphs drawn below and four randomly sorted functions

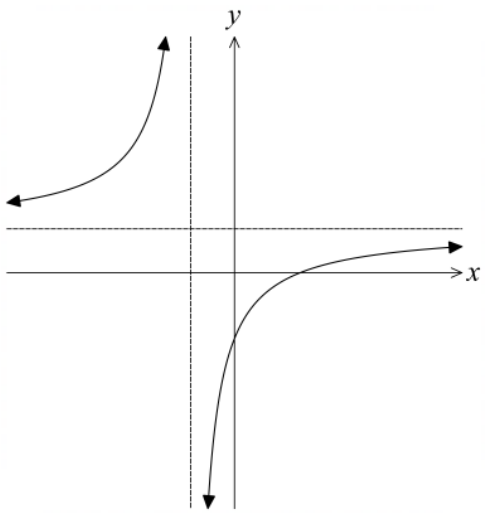
Graph α



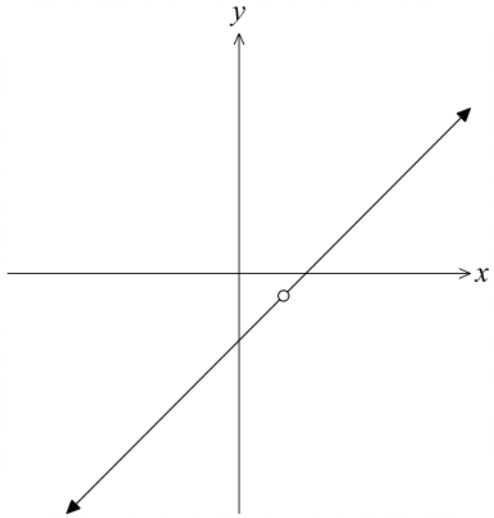
Graph β



Graph γ



Graph δ



Randomly sorted functions

I $\quad y = \dfrac{x}{x^2 - 16}$

II $\quad y = \dfrac{x^2 - 5x + 6}{x - 2}$

III $\quad y = \dfrac{4x}{x - 2}$

IV $\quad y = \dfrac{2x - 6}{x + 2}$

Which of the following shows the correct match up of a graph with its equation?

A. I δ, II β, III α, IV γ

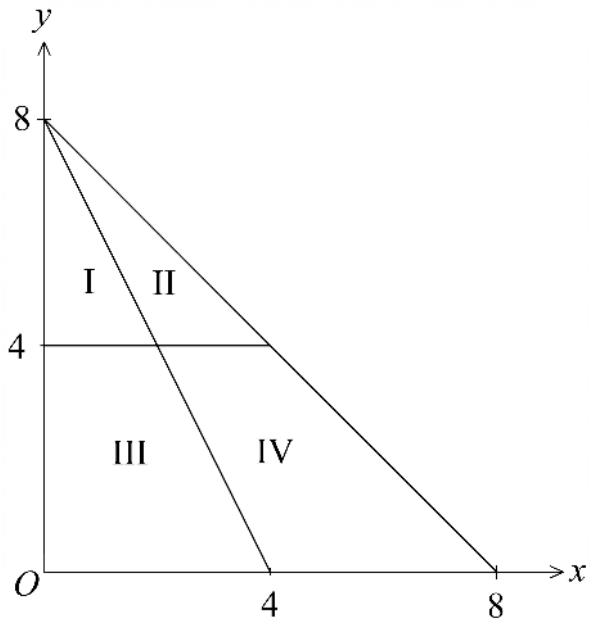
B. I δ, II β, III γ, IV α

C. I β, II δ, III α, IV γ

D. I β, II δ, III γ, IV α

7 Which of the regions in the figure may represent the solution of the following system of inequalities?

$$\begin{cases} y \geq 4 \\ x + y \leq 8 \\ 2x + y \geq 8 \end{cases}$$



A. Region I B. Region II C. Region III D. Region IV

8 Simplify $\frac{\cos 3\alpha}{\sin \alpha} + \frac{\sin 3\alpha}{\cos \alpha}$.

A. $\tan 2\alpha$ B. $\cot 2\alpha$ C. $2\tan 2\alpha$ D. $2\cot 2\alpha$

9 Below is a selection of the steps required to complete the proof of

$$\frac{2\left(\tan x-\cot x\right)}{\tan^2 x-\cot^2 x}=\sin 2x$$

The steps have been randomly sorted

1.	$\dfrac{2}{\dfrac{\sin^2 x}{\sin x\cos x}+\dfrac{\cos^2 x}{\sin x\cos x}}$
2.	$2\sin x\cos x$
3.	$\dfrac{2}{\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}}$
4.	$\dfrac{2}{\dfrac{1}{\sin x\cos x}}$

Which of the following shows correct order of the above steps in the proof?

A. 1, 2, 3, 4

B. 1, 3, 4, 2

C 3, 1, 4, 2

D 3, 2, 4, 1

10 Let α and β be such that $\pi<\alpha-\beta<3\pi$.

If $\sin\alpha+\sin\beta=-\dfrac{21}{65}$, then which of the following could be the value of $\cos\left(\dfrac{\alpha-\beta}{2}\right)$?

A. $-\dfrac{3}{\sqrt{130}}$ B. $\dfrac{3}{\sqrt{130}}$ C. $\dfrac{6}{65}$ D. $-\dfrac{6}{65}$

NESA Number										Q11	

Section II

41 marks
Attempt Questions 11–13
Allow about 1 hour and 15 minutes for this section

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

In Questions 11-13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Let $\cos\alpha = -\frac{2}{5}$, where $\sin\alpha > 0$ and $\sin\beta = \frac{4}{9}$, where $\tan\beta > 0$. **2**

Determine the exact value of $\cos(\alpha - \beta)$.

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(b) Solve the inequality $|3x - 5| < |x + 3|$. **2**

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Question 11 continued on page 8

Question 11 (continued)

(c) Let x be a real number satisfying $\left| x-2 \right| \leq 1$. **2**

Find the greatest value of $\left| x^2-6 \right|$.

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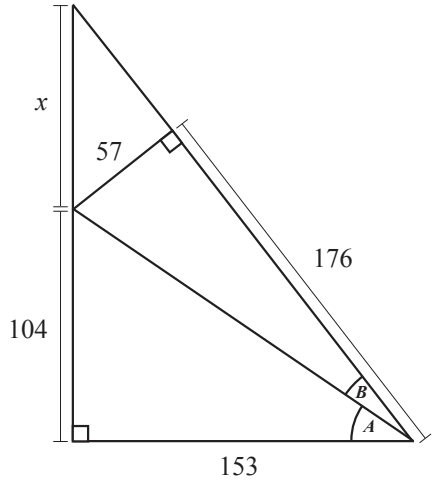
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(d) Using the diagram below and considering $\tan(A+B)$, find the exact value of x. **2**



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Question 11 continued on page 9

(e) (i) Solve $\frac{4-y}{y-2} \leq 0$ **2**

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(ii) Hence solve $\frac{4-2^x}{2^x-2} \leq 0$ **1**

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(f) Find an expression for $\tan 15°$ in surd form. **2**

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Question 11 continued on page 10

Question 11 (continued)

(g) Without using calculus, for $0° \le \theta \le 360°$, find the least value of $\dfrac{2+\sin\theta}{2-\sin\theta}$. **2**

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Additional space for Question 11 on page 11

Use this space to re-write any questions for Question 11.

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End of Question 11

NESA Number										Q12	

Question 12 (14 marks)

(a) What is the solution to the inequality $3-x \geq \frac{2}{x}$? **3**

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(b) Solve $5\cos\theta - 12\sin\theta = -3$ for $0 \leq \theta \leq 2\pi$ **3**
Leave your answers correct to 2 decimal places.

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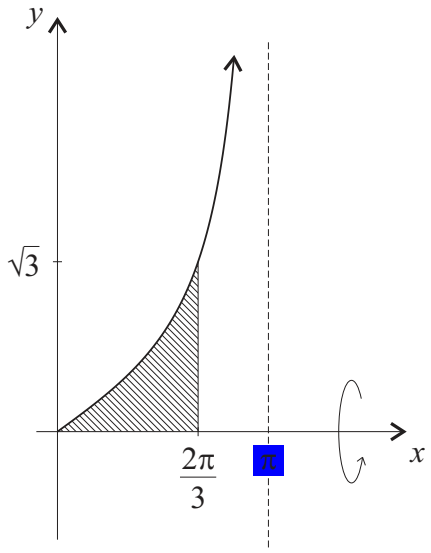
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Question 12 continued on page 13

(c) **3**



The diagram shows the region bounded by the curve $y = \tan\frac{x}{2}$, the x-axis and the line $x = \frac{2\pi}{3}$. Find the exact volume obtained by rotating this region about the x-axis.

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Question 12 continued on page 14

(d) Find $\int \sin 3x \cos 2x \, dx$ **2**

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(e) Let θ_1 and θ_2, where $\theta_1 > \theta_2$, be the solutions of the equation $6\cos\theta + 17\sin\theta = 18$ for $0 \le \theta \le 2\pi$. **3**

By letting $t = \tan\frac{\theta}{2}$, show that $\tan\left(\frac{\theta_1 - \theta_2}{2}\right) = \frac{1}{18}$.

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Additional space for Question 12 on page 15

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End of Question 12

NESA Number										Q13	

Question 13 (12 marks)

(a) Solve $3x^2 - 2x - 2 \leq |3x|$ **3**

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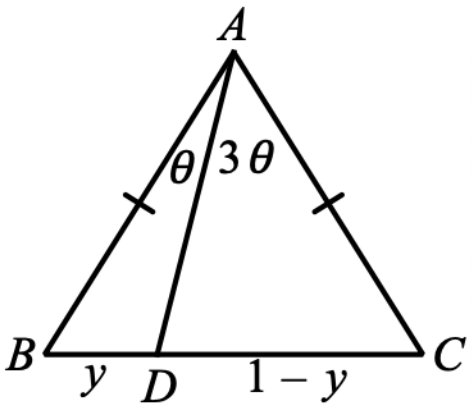
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Question 13 continued on page 17

Question 13 (continued)

(b)



In the diagram ABC is a triangle in which $AB = AC$ and $BC = 1$.
D is the point on BC such that $\angle BAD = \theta$, $\angle CAD = 3\theta$, $BD = y$ and $CD = 1 - y$.

(i) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **2**

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(ii) Hence show that $\sin^2\theta = \dfrac{4y-1}{4y}$. **2**

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Question 13 continued on page 18

(b) (iii) Hence show that $\frac{1}{4} < y < \frac{1}{2}$. **2**

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Question 13 continued on page 19

Question 13 (continued)

(c) If α, β and γ are the angles of a triangle, prove that **3**

$$\cos\alpha - \cos\beta + \cos\gamma = 4\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\gamma}{2} - 1.$$

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Additional space for Question 13 on page 20

Use this space to re-write any questions for Question 13.

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