Name: _____

Teacher: _____

Class: _____

FORT STREET HIGH SCHOOL

2013

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics Extension 1

Time allowed: 1 hour
(plus 5 minutes reading time)

Outcomes Assessed	Questions
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-5
Uses inductive reasoning in the construction of proofs	6a, 8b
Applies appropriate techniques to solve problems involving parametric representations	6b, 7b, 8a

Total Marks 33

Section I 5 marks
Multiple Choice, attempt all questions

Section II 28 marks
Attempt Questions 6-8

Section I	Total 5	Marks
Q1-Q5		
Section II	Total	Marks
Q6	/10	
Q7	/10	
Q8	/8	
	Percent	

General Instructions:

- Questions 6-8 are to be started in a new booklet.
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I

5marks
Attempt Questions 1-5

Use the multiple-choice answer sheet for Questions 1-5

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1 Mathematical induction is used to prove

$$1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n+1)(2n-1) \text{ for all positive integers } n\geq 1.$$

Which of the following has an incorrect expression for part of the induction proof?

(A) Step 1: To prove the statement true for $n=1$

$\text{LHS}=1^2=1 \qquad \text{RHS}=\frac{1}{3}\times 1\times(2\times 1+1)(2\times 1-1)=1$

Result is true for $n=1$

(B) Step 2: Assume the result true for $n=k$

$$1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{1}{3}(k+1)(2k+1)(2k-1)$$

(C) To prove the result is true for $n=k+1$

$$1^2+3^2+5^2+\ldots+(2k-1)^2+\big(2(k+1)-1\big)^2=\frac{1}{3}(k+1)\big(2(k+1)+1\big)\big(2(k+1)-1\big)$$

$$1^2+3^2+5^2+\ldots+(2k-1)^2+\big(2k+1\big)^2=\frac{1}{3}(k+1)\big(2k+3\big)\big(2k+1\big)$$

(D)

$$\begin{aligned}\text{LHS}&=1^2+3^2+5^2+\ldots+(2k-1)^2+\big(2(k+1)-1\big)^2\\&=\frac{1}{3}k(2k+1)(2k-1)+\big(2k+1\big)^2\\&=\frac{1}{3}(2k+1)\big(k(2k-1)+3\big(2k+1\big)\big)\\&=\frac{1}{3}(2k+1)\big(2k^2-k+6k+3\big)\\&=\frac{1}{3}(2k+1)\big(2k^2+5k+3\big)\\&=\frac{1}{3}(2k+1)(k+1)(2k+3)\\&=\text{RHS}\end{aligned}$$

2 A curve has parametric equations $x = t + 1$ and $y = 2t^2$.

What is Cartesian equation of this curve?

(A) $y = 2\sqrt{x-1}$

(B) $y = 2\sqrt{x+1}$

(C) $y = 2(x-1)^2$

(D) $y = 2(x+1)^2$

3 Victoria made an error proving that $3^{2n} - 1$ is divisible by 8 (where n is an integer greater than 0) using mathematical induction. Part of the proof is shown below.

Step 2: Assume the result true for $n = k$

$3^{2k} - 1 = 8P$ where P is an integer. Line 1

Hence $3^{2k} = 8P + 1$

To prove the result is true for $n = k + 1$

$3^{2(k+1)} - 1 = 8Q$ where Q is an integer. Line 2

$$\begin{aligned} \text{LHS} &= 3^{2(k+1)} - 1 \\ &= 3^{2k} \times 3^2 - 1 \\ &= (8P + 1) \times 3^2 - 1 \quad \text{Line 3} \\ &= 72P + 1 - 1 \quad \text{Line 4} \\ &= 72P \\ &= 8(9P) \\ &= 8Q \\ &= \text{RHS} \end{aligned}$$

In which line did Victoria make an error?

(A) Line 1 (B) Line 2

(C) Line 3 (D) Line 4

4 What is the equation of the chord of the parabola joining the points with parameters 1 and -3 on $x = 4t$ and $y = 2t^2$?

(A) $x - y - 3 = 0$

(B) $x + y - 3 = 0$

(C) $x - y - 6 = 0$

(D) $x + y - 6 = 0$

5 $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$. PQ is a focal chord of this parabola. PT and QT are parallel to the y-axis and x-axis respectively. What is the locus of T?

(A) $xy = 4a^2$

(B) $xy = 4a^3$

(C) $x^2y = 4a^2$

(D) $x^2y = 4a^3$

Section II

Attempt Questions 6– 8
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 6 – 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use a SEPARATE writing booklet.

(a) Use the principle of mathematical induction to prove that $5^n + 2(11)^n$ is a multiple of 3 for all positive integers n. **(5)**

(b) (i) Derive the equation of the normal to the parabola $x=2t$, $y=t^2$ at the point where $t = T$. **(3)**

(ii) Find the values (if any) of T for which the normal found in part (i) passes through $(0, 6)$. **(2)**

Question 7 (10 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x\to 0} \dfrac{\sin 3x}{2x}$ **(2)**

(b) Two points $P(2ap, ap^2)$ and $Q\ (2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

Given that the equation of the tangent to the parabola at P is given by $y - px + ap^2 = 0$

(i) Find the coordinates of the point of intersection T of the tangents to the parabola at P and Q. **(3)**

(ii) You are given that the tangents at P and Q in Part (ii) intersect at an angle of $60°$. **(1)**

Show that $p - q = \sqrt{3}\,(1 + pq)$

(iii) Hence find the locus of T. **(4)**

Question 8 (8 marks) Use a SEPARATE writing booklet.

(a) The equation of the chord of contact from an external point (x_o, y_o) is **(3)**

$$xx_o = 2a(y + y_o).$$

Show that the chord of contact from a point on the directrix of $x^2 = 4ay$ will always pass through the focus.

(b) Show by mathematical induction that $(1 + x)^n > 1 + nx + nx^2$ for all integers $n > 2$, given $x > 0$. **(5)**

End of paper