marks

Question 1 **(13 marks)**

a. Consider the function $f(x)=\frac{x}{4}+\frac{1}{x}$

i. Show that the function is odd. **1**

ii. Show that there is no value of x for which $f(x)=0$. **1**

iii. State the vertical asymptote of $y=f(x)$. **1**

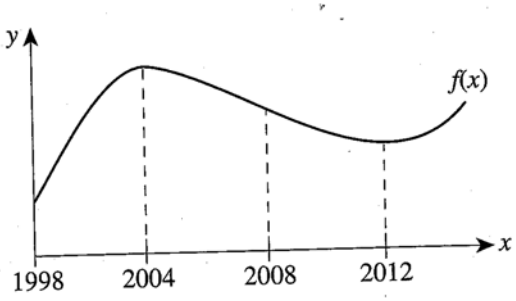
iv. Show that $(2,1)$ is one stationary point of $y=f(x)$ and find the coordinates of the other. **2**

v. Determine the nature of these stationary points. **1**

vi. Sketch the graph of $y=f(x)$, showing the above features as well as **all** asymptotes. **2**

vii. State the range of $y=f(x)$. **1**

b. The graph below represents the median house price in a particular Sydney suburb since 1998 and its predicted price for the next 8 years.



i. Draw a graph representing the first derivative of $f(x)$.

ii. What could you say about the sign of the second derivative at : **2**

- 2004
- 2008 **2**

Question 2 **(4 marks)**

Prove by induction that

4

$$\frac{2}{3}+\frac{10}{3}+8...+\frac{n}{3}(3n-1)=\frac{1}{3}n^2(n+1) \text{ for } n=1,2,3,...$$

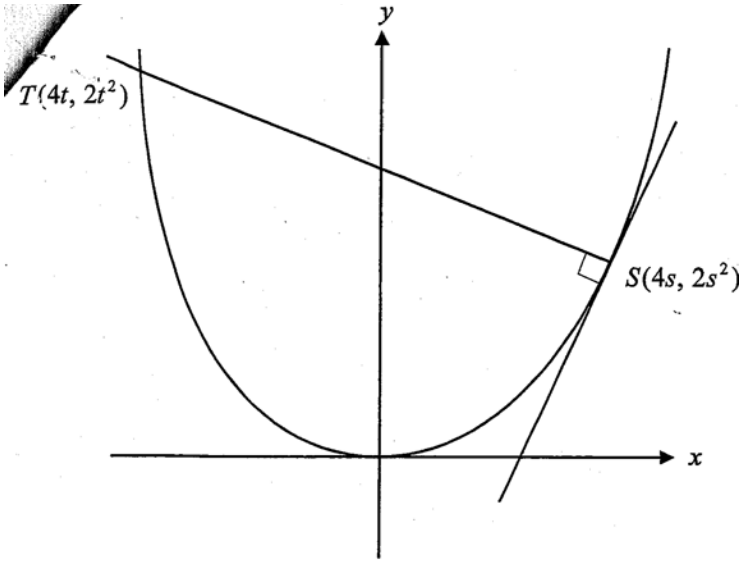
Question 3 **(5 marks)**

5

Prove by induction that $3^{3n}+2^{n+2}$ is divisible by 5 for all positive integers n.

Question 4 **(8 marks)**

a.



Consider the variable point $T(-2t,t^2)$ on the parabola $y=\frac{1}{4}x^2$.

i. Prove that the equation of the tangent at T is $y+tx+t^2=0$ **2**

ii. If A is the x intercept of the tangent at T, find the coordinates of A. **1**

iii. Find the coordinates of M, the midpoint of the interval TA. **1**

iv. Find the equation in Cartesian form of the locus of the point M given part (iii). **2**

b. The coordinates of a point P are given by $x = p + q$, $y = \frac{1}{2}\left(p^2 + q^2\right)$. Determine the cartesian equation of the locus of P , if $pq = 5$. **2**

Question 5 **(6 marks)**

The point $S\left(4s, 2s^2\right)$, $s > 0$, lies on the parabola as shown. The normal at s intersects the parabola at the point $T\left(4t, 2t^2\right)$, $t < 0$.

a. Find the Cartesian equation of the parabola. **1**

b. Show that the chord ST passes through the point $\left(0, 4 + 2s^2\right)$. **3**

c. Show that $t = -\left(s + \frac{2}{s}\right)$. **2**

d. Explain geometrically why $s \neq 0$. **1**