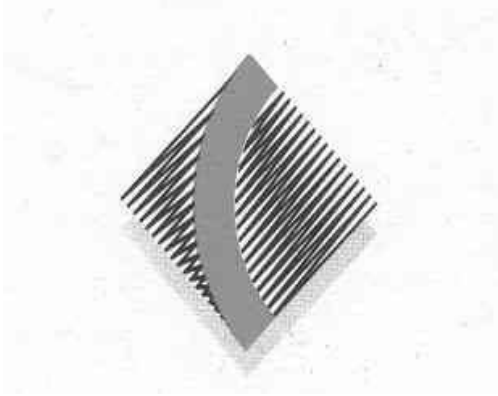
NAME_____

EW
KL
MK
KP
KW
AB

CLASS 12MTX___

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2015

YEAR 12

AT1

MATHEMATICS EXTENSION 1

Time allowed – 45 MINUTES

THERE IS NO READING TIME

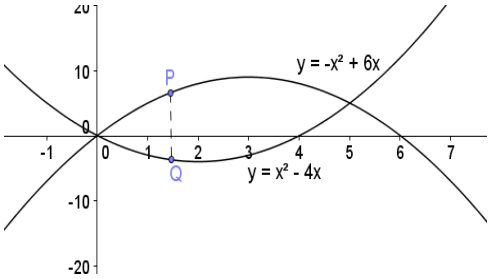
DIRECTIONS TO CANDIDATES:

- Attempt All Questions
- **Write** multiple choice answers on first page of solutions for question 1, 2, 3, 4
- Each long response question is to be commenced on a new page clearly marked with your name, class and Question 5, Question 6 etc on the top of the page.
- All necessary working should be shown in every question in section 2. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

Section 1 – MULTIPLE CHOICE (answers on page 1 of writing paper)

QUESTIONS (4 marks) MARKS

1. P, Q are points on the curves $y = x(6-x)$ *and* $y = x(x-4)$ respectively. The length (l) of PQ (PQ perpendicular to $x-$ axis) would be given by the expression: **1**



(A) $l = x(x-4) + x(6-x)$

(B) $l = x(6-x) - x(x-4)$

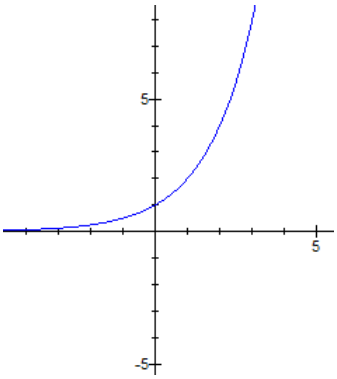
(C) $l = x(x-4) - x(6-x)$

(D) $l = x^2 - 5x$

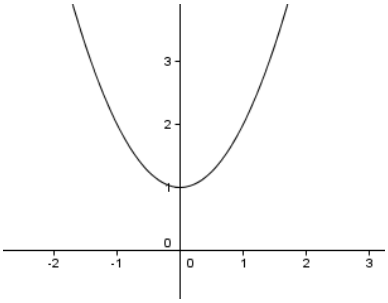
2. Which curve has the features : **1**

$$f'(x) > 0 \text{ and } f''(x) > 0$$

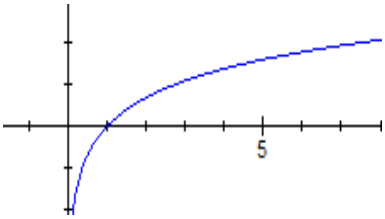
(A)



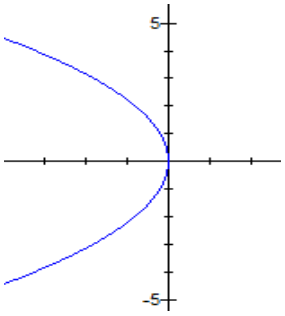
(B)



(C)



(D)



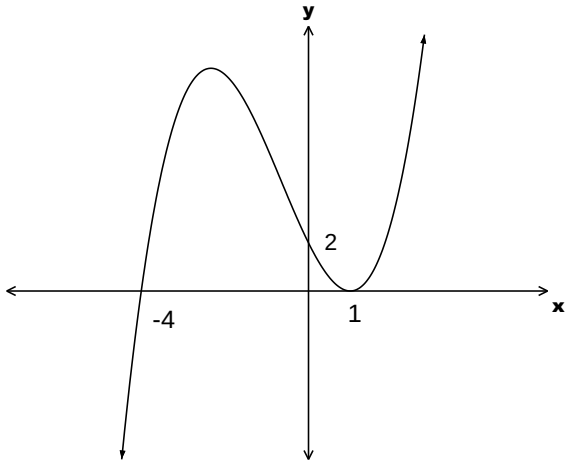
3 The sum of the first 21 terms of the arithmetic series $3\frac{1}{2} + 1 - 1\frac{1}{2}\ldots$ is **1**

(A) -451.5 (B) -488.25

(C) 598.5 (D) 57

MULTIPLE CHOICE CONTIUED

4. The diagram shows part of the graph of a cubic function **1**



What is the equation of this graph?

(A) $y = \frac{1}{2}(x-1)^2(x+4)$

(B) $y = (x-1)^2(x+4)$

(C) $y = \frac{1}{2}(x-4)^2(x+1)$

(D) $y = 2(x-1)^2(x+4)$

Section 2 Short answer questions

Question 5 (8 marks) (start on a new writing page) MARKS

(a) Find the first two terms of the arithmetic sequence in which the 4th term is 21 and the 7th term is 39. **2**

(b) The sum to infinity of a geometric series is 27, while the sum of the first three terms is 19. Find the first term and the common ratio. **2**

(c) In drilling an oil well 400m deep, the cost is \$150 for the first metre and increases \$20 for each additional metre. What is the cost of drilling the well? **2**

(d) Find two unequal numbers p, q such that 1, p, q … form an arithmetic sequence and 1, q, p ... form a geometric sequence. **2**

Question 6 (10 marks) (start on a new writing page) MARKS

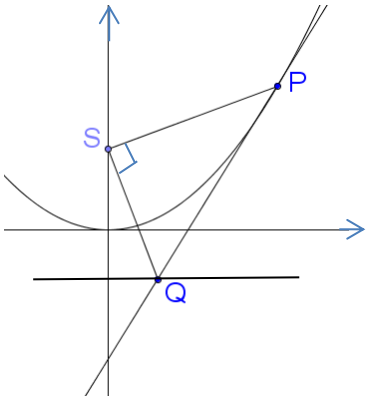
(a) If $x = 6t$ and $y = 3t^2$
then the cartesian equation would be
(A) $y^2 = 12x$ (B) $x^2 = 12y$ (C) $x^2 = \frac{y}{12}$ (D) $x^2 = 6ay$ **1**

(b) Tangents to the parabola $x = 2at, y = at^2$, from the points P and Q where $t = p$ and $t = 2p$ respectively, meet at T.

(i) Show that the point of intersection of the tangents is $(3ap, 2ap^2)$ **2**

(ii) Find the locus of the point of intersection of these tangents. **2**

(c)



Consider the parabola $x^2 = 4ay$.

If the equation of the tangent to $x^2 = 4ay$ at the point $P(2ap, ap^2)$ is $y = px - ap^2$ then

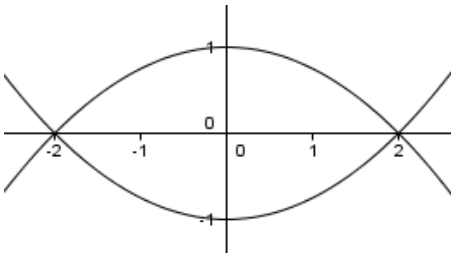
(i) The tangent at P meets the directrix at Q. Find the coordinates of Q **1**

(ii) Prove that PQ subtends a right angle at the focus. **2**

(d) The focus of each of the parabolas

$$y = \frac{1}{4}x^2 - 1 \text{ and } y = -\frac{1}{4}x^2 + 1$$

is (0,0) as shown in the diagram.



A closed double parabolic reflector has cross section through the centre of symmetry O as shown. A light source is placed at O; explain carefully what will happen to rays emitted from the source. **2**

Question 7 (11 marks) (start on a new writing page) MARKS

(a) Consider the function $f(x) = \frac{x^2-1}{x^2+1}$

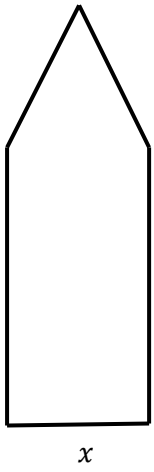
(i) Find asymptotes **1**

(ii) Find the coordinates of any stationary point(s) and determine the nature **2**

(iii) Find where the curve crosses the x-axis **1**

(iv) Sketch the curve $= f(x)$, showing all essential features. **2**

(b) The figure represents a large window made up of a rectangle and an equilateral triangle. The beading around the outside of the window is 12m long.



(i) Show that the expression for the area of the figure is **3**

$$A = \frac{24x + x^2(\sqrt{3}-6)}{4}$$

(ii) Hence calculate the base (x) of the window so that it allows the maximum light through (answer to 1 decimal place) **2**

END of EXAMINATION