Question 1 [START A NEW PAGE] **Marks**

(a) Find the primitive of the function $y = 3x^3 - 1$ **1**

(b) Differentiate: e^{x^2} **1**

(c) Find:

(i) $\int_0^6 (x)(\sqrt[3]{x^4})\, dx$ **2**

(ii) $\int -\sec^2(3 - 2x)\, dx$ **2**

(d) The coordinates of A, B and C are $(4,3)$, $(0,-3)$ and $(4,0)$ respectively.

(i) Show that the line BC has equation $3x - 4y - 12 = 0$ **2**

(ii) Plot the points A, B and C, and draw the intervals OA and BC, on a number plane, where O is the origin **1**

(iii) Prove that $OACB$ is a parallelogram **3**

(iv) Find the area of parallelogram $OACB$ **1**

(e) A circle has diameter 10 cm. Find the area of the minor segment that subtends an angle of $\frac{\pi}{7}$ radians at the centre of the circle. **2**

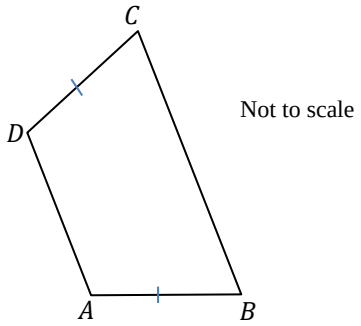
Question 2 [START A NEW PAGE] **Marks**

(a) Find:

(i) $\int_1^3 \frac{25x^4-1}{5x^2+1}\,dx$ **2**

(ii) $\int_{-\frac{\pi}{4}}^{\pi} \sin\left(\frac{x}{2}\right) dx$ **2**

(b)



Not to scale

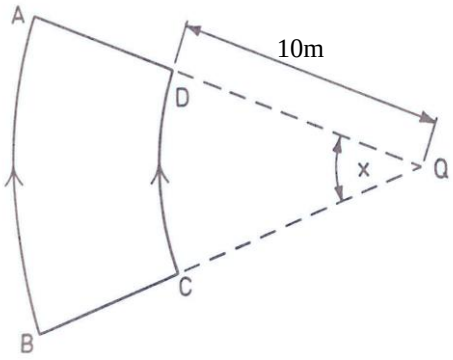
In quadrilateral $ABCD$ above, $AB = CD$ and $\angle ABC = \angle BCD$.

Prove $\angle ABD = \angle DCA$. **4**

(c) (i) Sketch the function $y = \frac{2}{x}$ **1**

(ii) Using the trapezoidal rule with three function values, find the approximate area of the region formed between the curve $y = \frac{2}{x}$, the x axis and the lines $x = -1$ and $x = -4$ **3**

(d)



In the diagram above, AB and CD are circular arcs that subtend an angle x radians at the centre Q, where $0 < x < \pi$, and AQ, BQ are radii. The radius of the outer circle is 15 metres and $DQ = 10$ metres.

(i) Find the length of each arc, AB and DC in terms of x **2**

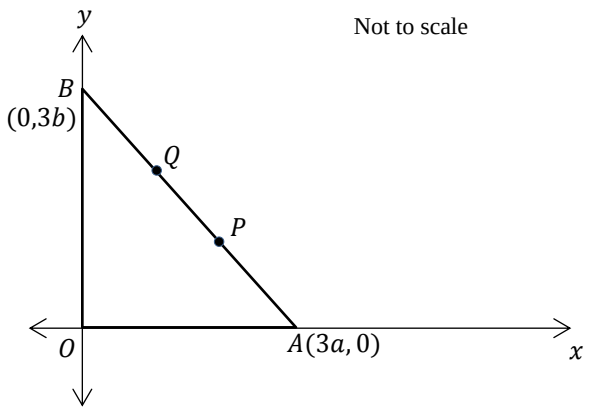
(ii) Given $x = \frac{3\pi}{10}$, find the perimeter of the figure $ABCD$. **1**

Question 3 [START A NEW PAGE] **Marks**

(a) (i) Sketch the curve $y = \sqrt{4 - x^2} + 4$ **2**

(ii) The curve described above is rotated about the y axis between $y = 4$ and $y = 6$. Find the volume of the solid formed by this rotation **2**

(b)



In the number plane diagram above, ΔBOA is right-angled with one vertex at the origin and its hypotenuse stretching from $A(3a, 0)$ to $B(0,3b)$.

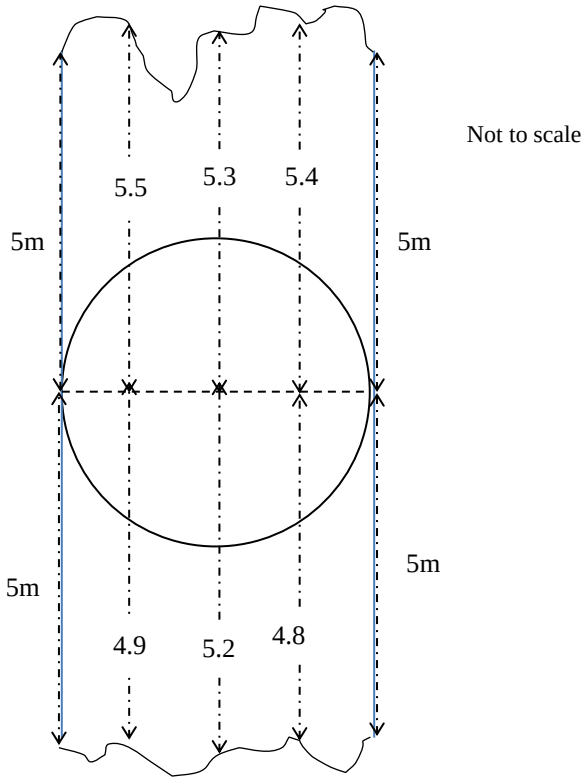
$BQ = QP = PA$.

(i) Show that P and Q have coordinates $(2a, b)$ and $(a, 2b)$ respectively **3**

(ii) Hence prove $OP^2 + PQ^2 + OQ^2 = \frac{2}{3} \times AB^2$ **3**

(c) A circular pillar sticks out of a flat-bottomed pond that used to be rectangular. The centre of the pillar is in the middle of the rectangle and the edges of the pillar just touch the sides of the pond.

The pond was originally 10 metres long, 4 metres wide and 5 metres deep, but now its ends are broken and irregularly shaped. The distance from the middle of the pillar to the broken ends of the pond was measured in 1 metre intervals as shown below.



(i) Calculate the area of the circular cross-section of the pillar **1**

(ii) Hence, using Simpson's rule and the measured distances, determine, approximately, the current capacity of the broken-ended pond. Give your answer in litres, correct to 3 decimal places **4**

Question 4 [START A NEW PAGE] **Marks**

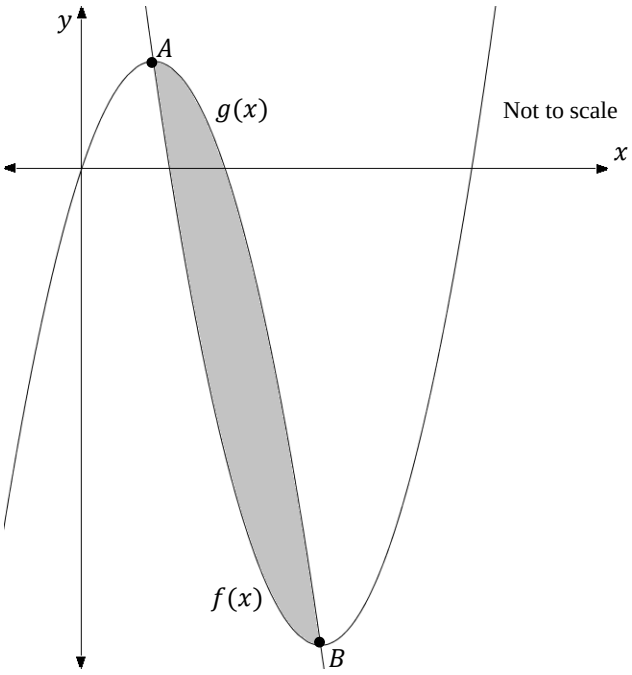
(a) A solid of revolution is formed by rotating about the x axis the portion of the curve

$$y = \frac{1}{\sqrt[4]{3-x}}$$

between the ordinates $x = -5$ and $x = 2$.

Find the exact volume of this solid. **3**

(b)



The graph above shows the curves $f(x) = x^2 - 10x + 15$ and $g(x) = 3x - x^2$.

(i) Find the exact values of the x coordinates of the points marked A and B **2**

(ii) Calculate the exact area of the shaded region **2**

(c) A function is defined as follows:

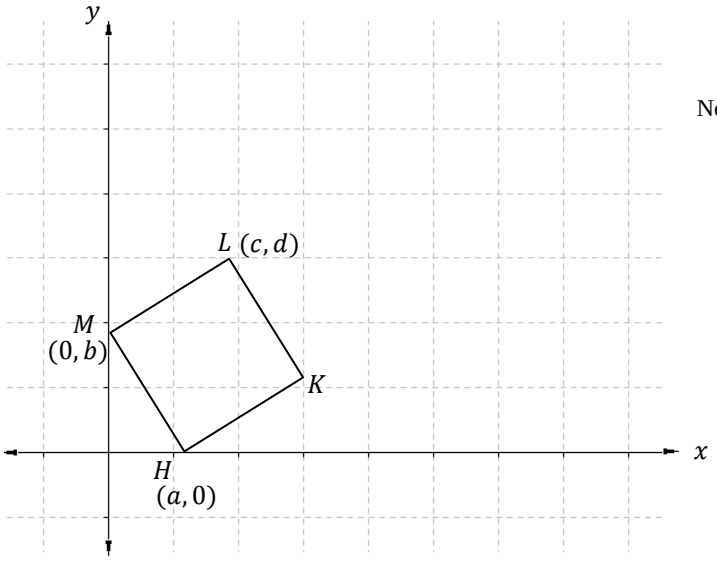
$$f(x) = \begin{cases} |x+3| - 2, & x \leq 1 \\ |x-5| - 2, & x > 1 \end{cases}$$

(i) Graph this function on a number plane **3**

(ii) Evaluate

$$\int_{-5}^{7} f(x)\mathrm{d}x$$ **1**

(d)



Not to scale

The square $HKLM$ in the figure above has two opposite vertices $H(a, 0)$ and $L(c, d)$ respectively, where $c > a$.

(i) Find the midpoint, X, of HL **1**

(ii) Hence, by finding the equation of the line through M and X, show **3**

$$b = \frac{c^2 + d^2 - a^2}{2d}$$

END OF EXAM