Student Number: _____ Teacher Name: _____

PENRITH SELECTIVE HIGH SCHOOL

MATHEMATICS DEPARTMENT

Year 11 Yearly Examination
2025
Mathematics Extension 1

General Instructions:

- Reading time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided with this examination paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No correction tape/white out allowed.

Total marks: 60

Section I – 10 marks (pages 1–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 50 marks (pages 5–10)

- Attempt Questions 11–14
- Allow about 1 hour and 15 minutes for this section

	Multiple Choice	Q11	Q12	Q13	Q14	TOTAL
Further work with Functions	1, 5 /2	a, b /6				/8
Polynomials	3, 6 /2	d /3		a, b /6		/11
Inverse Trigonometric Functions	2, 10 /2	c /3	a, d /6			/11
Further Trigonometric Identities	8, 9 /2		b /3	d /3	b /4	/12
Rates of Change	4, 7 /2		c /3	c /4	a, c /9	/18
TOTAL	/10	/12	/12	/13	/13	/60

Section I
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1–10.

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1 Let $f(x) = k\log_2(x), k \in R$.

Given that $f^{-1}(1) = 8$, what is the value of k?

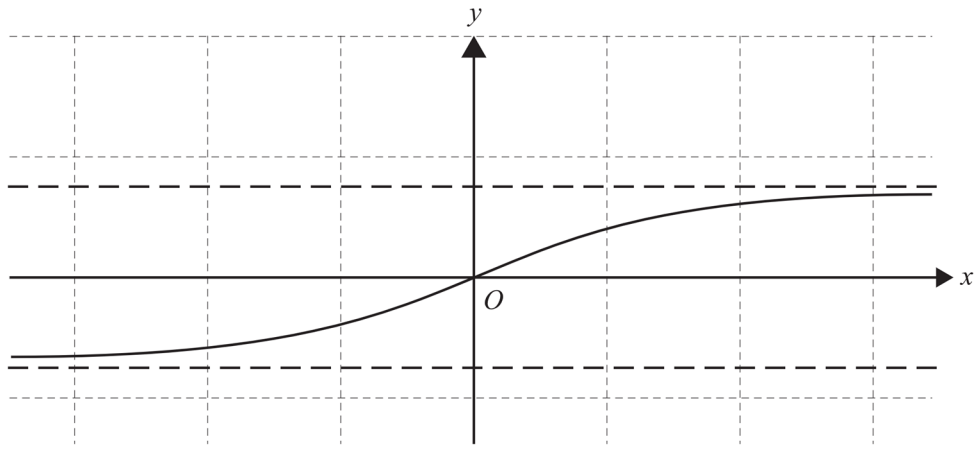
A. 0

B. $\frac{1}{3}$

C. 3

D. 8

2 Part of the graph of $y = \frac{1}{2}\tan^{-1} x$ is shown below.



The equations of its asymptotes are

A. $y = \pm\frac{3}{4}$

B. $y = \pm 1$

C. $y = \pm\frac{\pi}{2}$

D. $y = \pm\frac{\pi}{4}$

3 If $x + a$ is a factor of $7x^3 + 9x^2 - 5ax$, what are the possible values of a?

A. $-2, -1$

B. $-1, 1$

C. $1, 2$

D. $0, 2$

4 A tram is travelling along its track. Its displacement from a fixed point at time t seconds is x metres. At a certain moment, the velocity is $v = -6$ m/s and the acceleration is $a = -2$ m/s^2.

Which statement best describes the motion of the tram at that instant?

A. The tram is moving to the left with increasing speed.

B. The tram is moving to the left with decreasing speed.

C. The tram is moving to the right with increasing speed.

D. The tram is moving to the right with decreasing speed.

5 The position of a particle moving in the Cartesian plane, at time t, is given by the parametric equations

$$x = \frac{6t}{t+1} \text{ and } y = \frac{-8}{t^2+4}, \text{ where } t \geq 0.$$

What is the slope of the tangent to the path of the particle when $t = 2$?

A. $-\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

6 The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

A. $k = 6$

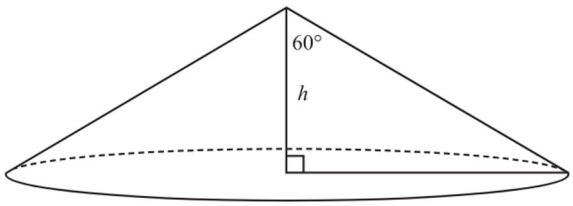
B. $k > 6$ or $k < -2$

C. $-2 \leq k \leq 6$

D. $k = -2$

7 Sugar falls from a chute to form a pile in the shape of a right circular cone with semi-vertex angle $60°$. Sugar is added to the pile at a rate of $1.5\ \text{m}^3$ per minute.

What is the rate at which the height h metres of the pile is increasing, in metres per minute, when the height of the pile is 0.5 m, correct to two decimal places?



A. 0.31 metres per minute

B. 0.64 metres per minute

C. 3.53 metres per minute

D. 3.82 metres per minute

8 The expression $1 - \frac{4\sin^2\theta}{\tan^2\theta + 1}$ simplifies to

A. $\cos^2(2\theta)$

B. $2\sin(2\theta)$

C. $2\sin^2(2\theta)$

D. $1 - 2\cos^2(2\theta)$

9 The angle θ satisfies $\sin\theta = -\frac{7}{25}$ and $\pi < \theta < \frac{3\pi}{2}$.

What is the exact value of $\cos 2\theta$?

A. $-\frac{336}{625}$

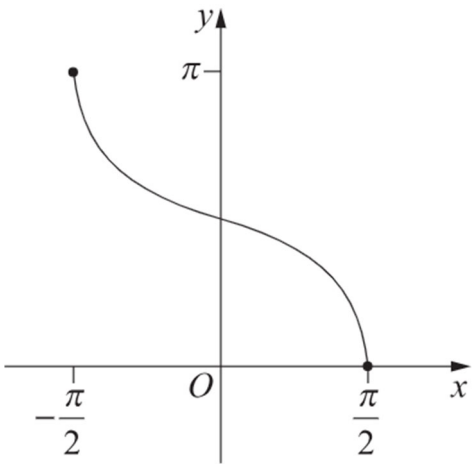
B. $\frac{336}{625}$

C. $-\frac{527}{625}$

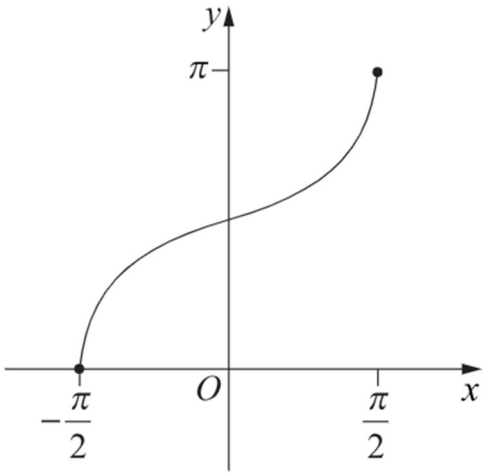
D. $\frac{527}{625}$

10 Which graph best represents $y = \cos^{-1}(\sin x)$, for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$?

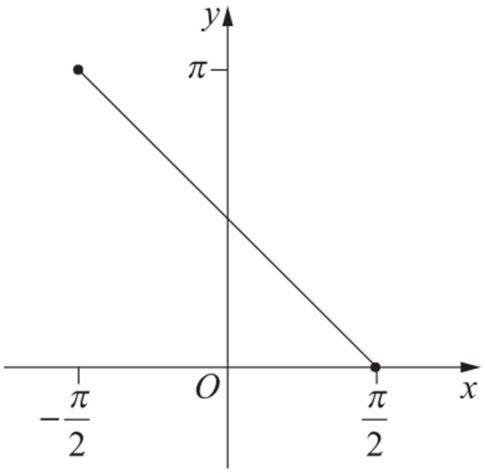
A.



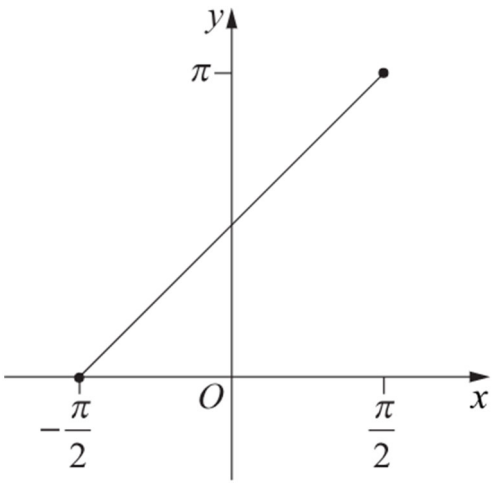
B.



C.



D.



Section II

50 marks
Attempt Questions 11–14
Allow about 1 hour and 15 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Use a separate Writing Booklet for Question 11.

(a) Solve the inequality $x^2 \leq |2x - 1|$ for x. **3**

Express your answer in interval notation.

(b) A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $x \neq -5$. **3**

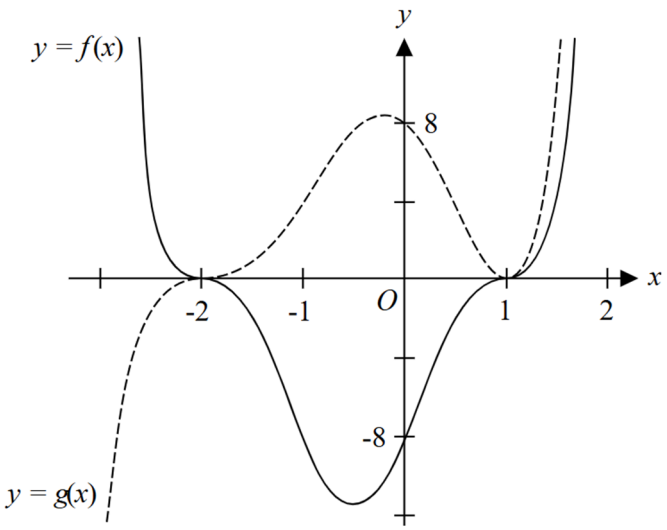
Find an expression for the inverse function $f^{-1}(x)$ and hence find the domain of the inverse function $f^{-1}(x)$.

(c) Find $\sin\theta$ given that $\theta = \cos^{-1}\left(\frac{12}{13}\right) + \tan^{-1}\left(\frac{3}{4}\right)$, θ is acute. **3**

Question 11 continues on page 6

Question 11 (continued)

(d) The graph of $f(x) = (x-1)^3(x+2)^3$ and $g(x) = (x-1)^2(x+2)^3$ are shown on the axes below.



The two graphs intersect at three points, $(-2, 0), (1, 0)$ and (a, b). The point (a, b) is not shown in the diagram.

(i) Find the values of a and b. **2**

(ii) Hence, find the values of x such that $f(x) > g(x)$. **1**

End of Question 11

Question 12 (12 marks) Use a separate Writing Booklet for Question 12.

(a) Consider the function **3**

$$f(x) = \sin^{-1}\left(\frac{x}{2}\right) + \frac{3}{\sqrt{25x^2 - 1}}$$

Find the largest set of values of $x \in R$ for which $f(x)$ is defined.

(b) Prove the following identity **3**

$$\frac{\sin 2x - \sin x}{\cos 2x - \cos x + 1} = \tan x$$

(c) A spherical water purification tablet of radius r metres evaporates at a rate proportional to its surface area. The rate of change of its volume V of the tablet is given by

$\frac{dV}{dt} = -2 \times 10^{-4} A$, where t is time in seconds and A is the surface area of the tablet.

(i) Show that $\frac{dr}{dt}$ is a constant. **1**

(ii) How long does it take for the water purification tablet of volume 10^{-6} m^3 to completely evaporate? **2**

(d) State the domain and range of $y = \cos^{-1}(1 - 2x)$. **3**

Hence, sketch the graph of $y = \cos^{-1}(1 - 2x)$ over its domain.

Label the end points with their coordinates.

End of Question 12

Question 13 (13 marks) Use a separate Writing Booklet for Question 13.

(a) The polynomial $P(x) = 3x^3 - px^2 + qx + 48$ has a double root at $x = 4$. **3**

Find the values of p and q and hence find the other root of $P(x)$.

(b) If $x^3 - 8x^2 + kx - 12 = 0$ has one root equal to the sum of the other two, find the value of k. **3**

(c) A boiled egg at 96℃ is placed to cool in a bowl of water at 15℃. After 5 minutes the egg's temperature is 40℃. (Assume that the temperature of the water does not change.)

(i) Find the temperature of the egg when $t = 10$. **2**

(ii) How long does it take for the egg to reach a temperature of 30℃ ? **2**

(d) Given that $8\cos\theta - \sin\theta = 4$, find the possible values of $\tan\left(\frac{\theta}{2}\right)$. **3**

End of Question 13

Question 14 (13 marks) Use a separate Writing Booklet for Question 14.

(a) A commercial egg farm has 16 000 chickens, the number of chickens P infected with a recent outbreak of bird flu at time t years is given by

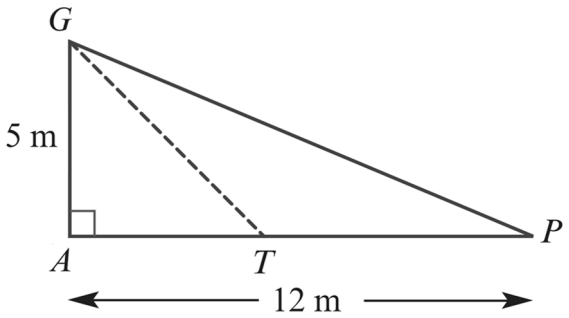
$$P = \frac{16\,000}{1 + ae^{-1000t}}, \text{where } a \text{ is a constant.}$$

(i) Show that, eventually, all chickens will be infected by the bird flu. **1**

(ii) Suppose that when time $t = 0$, exactly one chicken was infected with bird flu. After how many days will 8 000 chickens be infected? **2**

(iii) Show that $\frac{dP}{dt} = \frac{P}{16}(16\,000 - P)$. **2**

(b) In a right-angled triangle $GAP, AP = 12$ m and $GA = 5$ m.
The point T on AP is such that $\angle AGT = \angle TGP = x$.



(i) Find the exact value of $\tan x$. **3**

(ii) Hence, find the exact value of AT. **1**

Question 14 continues on page 10

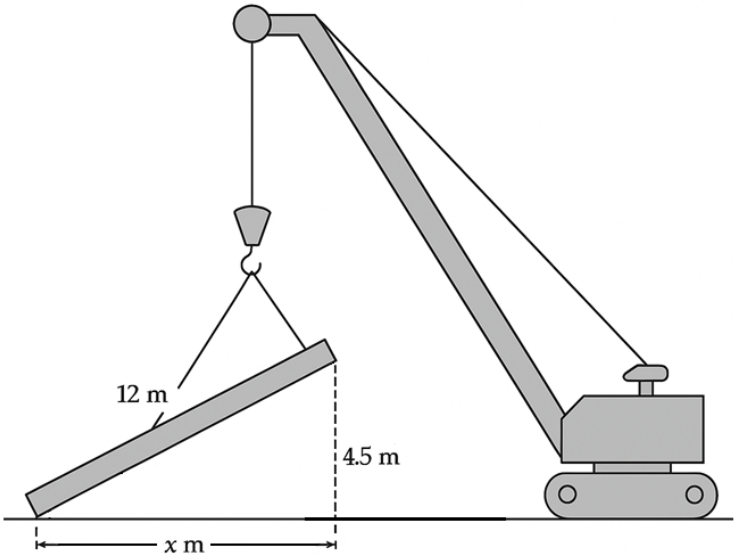
Question 14 (continued)

(c) A rigid iron girder of length 12 m has one end dragged along level ground while the other end is lifted vertically by a crane.

Let $h(t)$ be the height (in metres) of the lifted end, $x(t)$ be the horizontal distance (in metres) between the ground contact point and the vertical under the lifted end. The girder remains straight and taut.

The crane lifts at a constant rate $\frac{dh}{dt} = 0.06 \text{ ms}^{-1}$.

At the instant $h = 4.5$ m.



(i) Show that $\frac{dx}{dt} = -\frac{h}{x}\frac{dh}{dt}$ and find $\frac{dx}{dt}$. **3**

(ii) At the instant when the lifted end of the girder is at a height of 4.5 m, the midpoint M of girder is moving. A peg P is fixed on the ground 5 m to the left of the ground contact point. **1**

Explain, without full calculation, whether the distance $D = MP$ is increasing or decreasing at that instant. Support your answer with reference to the signs of the horizontal and vertical components of motion.

END OF PAPER