Name: _____

KNOX GRAMMAR SCHOOL **Teacher:** _____

2018

Year 11 Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- NESA-approved calculators may be used
- In Questions 11 – 14, show all relevant mathematical reasoning and/or calculations

Teachers:
Mr Willcocks (Examiner)
Ms Yun
Mr Mulray
Mr Meli

the examination room

Total marks – 70

Section I: Pages 3 – 5

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 6 – 11

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

This paper MUST NOT be removed from
Write your full name and your teacher's name on the front cover of each writing booklet

Number of Students in Course: 106

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 Which of the following is an expression for $\dfrac{9^n - 6^n}{9^n - 4^n}$?

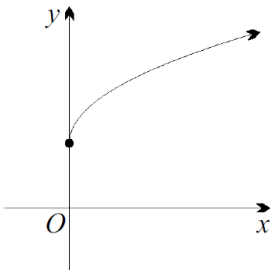
(A) $\dfrac{6^n}{4^n}$

(B) $\dfrac{3^n}{5^n}$

(C) $\dfrac{3^n}{3^n - 2^n}$

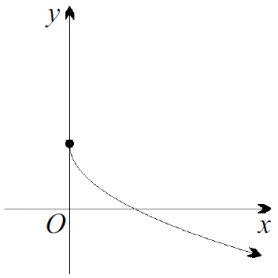
(D) $\dfrac{3^n}{3^n + 2^n}$

2

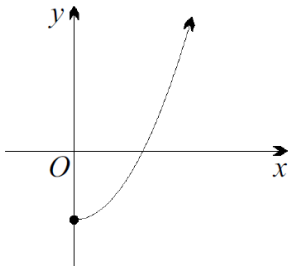


The diagram above shows the graph of $y = f(x)$. Which diagram below shows the graph of $f^{-1}(x)$?

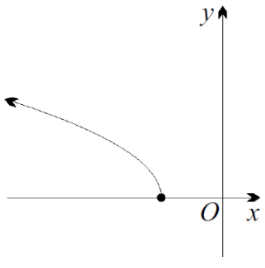
(A)



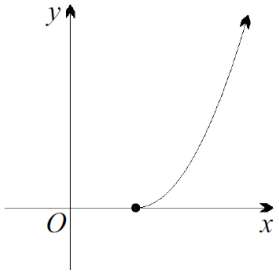
(B)



(C)

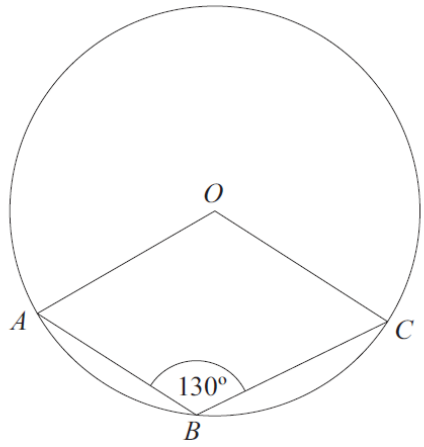


(D)



Section I continued on the following page

3 The points A, B and C lie on a circle with centre O. If $\angle ABC = 130°$, what is the size of the obtuse $\angle AOC$?



(A) $50°$ (B) $65°$

(C) $100°$ (D) $260°$

4 Which of the following is the correct simplified expression for differentiating $f(x) = \dfrac{1}{x}$ from first principles.

(A) $f'(x) = \lim\limits_{h \to 0} \dfrac{-1}{x(x+h)}$ (B) $f'(x) = \lim\limits_{h \to 0} \dfrac{x+h-x}{h}$

(C) $f'(x) = \lim\limits_{h \to 0} \dfrac{\frac{1}{x} - \frac{1}{x+h}}{h}$ (D) $f'(x) = \lim\limits_{h \to 0} \dfrac{h}{x+h-x}$

5 T divides PQ internally in the ratio $2:1$, R divides PQ externally in the ratio $2:1$. In what ratio does Q divide RT?

(A) $3:1$ (B) $1:3$

(C) $3:4$ (D) $4:3$

6 A curve has parametric equations $x = at$ and $y = \frac{1}{2}\left(a + at^2\right)$. What is the Cartesian equation of the curve?

(A) $x^2 = 4ay$ (B) $x^2 = 2a\left(y - \frac{1}{2}a\right)$

(C) $x^2 = 2a(y - a)$ (D) $x^2 = 2y - a$

Section I continued on the following page

7 For $y=\sqrt{1-f(x)}$, $\dfrac{dy}{dx}$ is equal to:

(A) $\dfrac{2f'(x)}{\sqrt{1-f(x)}}$ (B) $\dfrac{-1}{2\sqrt{1-f(x)}}$

(C) $\dfrac{1}{2}\sqrt{1-f'(x)}$ (D) $\dfrac{-f'(x)}{2\sqrt{1-f(x)}}$

8 Two straight lines have gradients $m-1$ and $m+1$. The acute angle between the lines is θ. Which of the following is an expression for $\tan\theta$?

(A) $\dfrac{-2}{m^2}$ (B) $\dfrac{2}{m^2}$

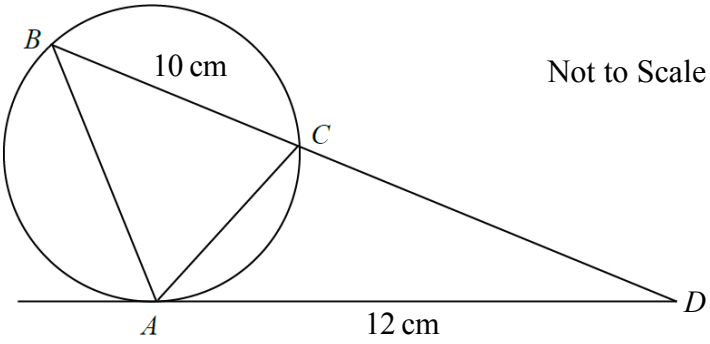
(C) $\left|\dfrac{2m}{2-m^2}\right|$ (D) $\left|\dfrac{2}{2-m^2}\right|$

9 What is the equation of the chord of contact to the parabola $x^2=8y$ from the point $(3,-2)$?

(A) $x-8y=0$ (B) $2x-3y+6=0$

(C) $3x-4y+8=0$ (D) $x+2y-5=0$

10 ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D where $BC=10$ and $AD=12$. What is the length of CD?



Not to Scale

(A) 7 cm (B) 8 cm

(C) 9 cm (D) 10 cm

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$. Showing all working, find the values of a and b. **3**

(b) Find the domain of the function $f(x) = \sqrt{9 - x^2} + \dfrac{1}{x+3}$. **2**

(c) (i) Without the use of calculus, sketch (using half a page), the polynomial $y = x(x+1)(x-4)$ showing all the intercepts with the axes. **2**

(ii) Hence, or otherwise, solve the inequation $\dfrac{x(x+1)}{(x-4)} \geq 0$. **2**

(d) Suppose that the equation $x^2 + 5x - 1 = 0$ has roots α and β. Without solving the equation, find the values of:

(i) $(\alpha - \beta)^2$ **2**

(ii) $|\alpha - \beta|$ **1**

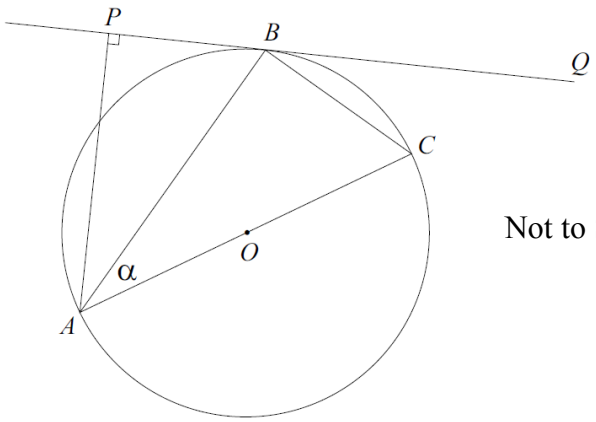
(e) The angle between the line $4x + 3y = 8$ and the line $ax + by + c = 0$ is $45°$. Find the possible values of the ratio $a : b$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) The diagram below shows the points A, B and C lying on the circumference of a circle, where AC is a diameter. The line AP is perpendicular to the tangent at B. Let $\angle BAC = \alpha$.



Not to Scale

Prove that BA bisects $\angle PAC$. **3**

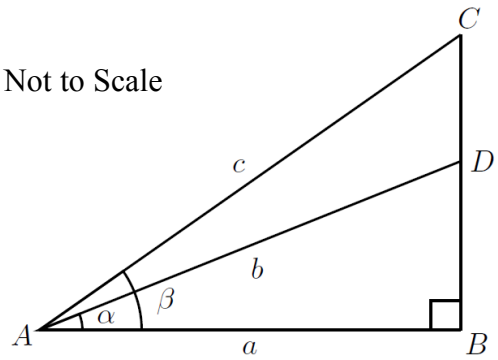
(b) A series is given by $1+\dfrac{1-p}{p}+\left(\dfrac{1-p}{p}\right)^2+\ldots$, where p is positive.

(i) For what values of p does a limiting sum exist. **2**

(ii) Find the sum to infinity in terms of p. **1**

(c) In the diagram below, $\angle BAD = \alpha$, $\angle BAC = \beta$ and sides AB, AD and AC have lengths a, b and c respectively.

Not to Scale



(i) Write down an expression for the area of ΔACD in terms of b, c, α and β. **1**

(ii) Hence show that $\sin(\beta-\alpha)=\sin\beta\cos\alpha-\sin\alpha\cos\beta$. **3**

Question 12 continued on the following page

Marks

(d) Andrew needs to borrow \$700 000 to purchase an apartment. He has calculated that he is able to afford monthly repayments of \$6000 per month. The loan is to be repaid in equal monthly instalments of \$$M$ over 30 years. Reducible interest is charged at 9·6% per annum and is calculated monthly. Let $\$A_n$ be the amount owing after n months.

(i) Write down an expression for A_1 and show that the amount owing after 2 months is given by $A_2 = 700\,000(1.008)^2 - M(1+1.008)$ **2**

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(ii) Hence show that $A_n = 700\,000(1.008)^n - 125M(1.008^n - 1)$. **1**

(iii) Calculate the monthly instalment \$$M$, required to pay off the loan at the end of 30 years correct to the nearest dollar, and determine whether or not Andrew will be able to purchase this unit. **2**

End of Question 12

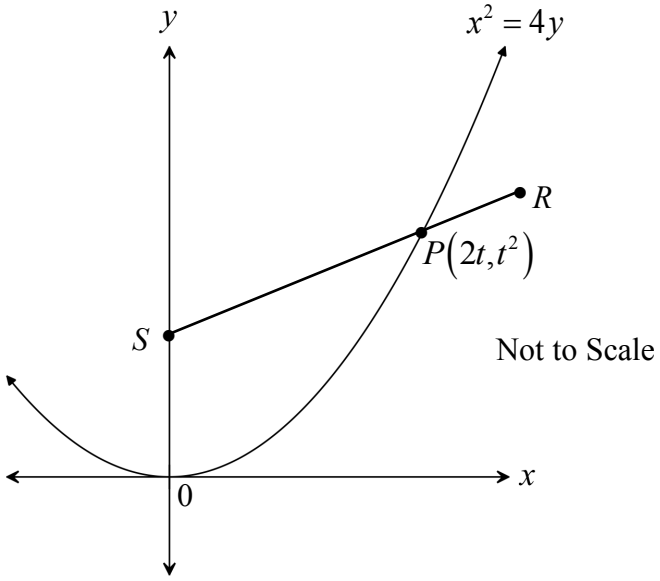
Question 13 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) Consider the series $\log\frac{a^3}{\sqrt{b}}+\log\frac{a^3}{b}+\log\frac{a^3}{b\sqrt{b}}+...$

Prove that it is arithmetic and state the common difference. **2**

(b) $P\left(2t,t^2\right)$ is a point on the parabola $x^2=4y$, with focus S. The point R divides the interval SP externally in the ratio $3:1$.



(i) Show that the coordinates of R are $\left(3t,\frac{3t^2-1}{2}\right)$. **2**

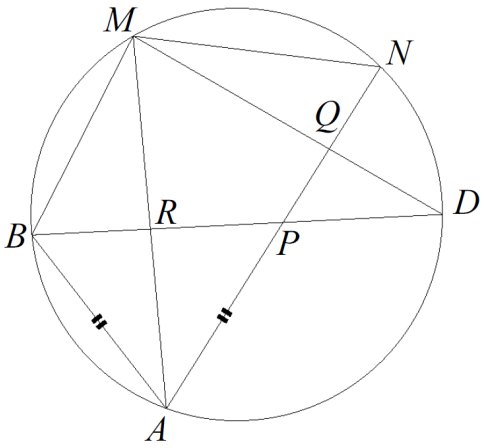
(ii) Show that as P moves on the parabola $x^2=4y$, the locus of R is given by $x^2=6y+3$. **2**

(iii) Draw a sketch of the locus of R in part (ii) labelling the coordinates of the focus, vertex and directrix. **2**

Question 13 continued on the following page

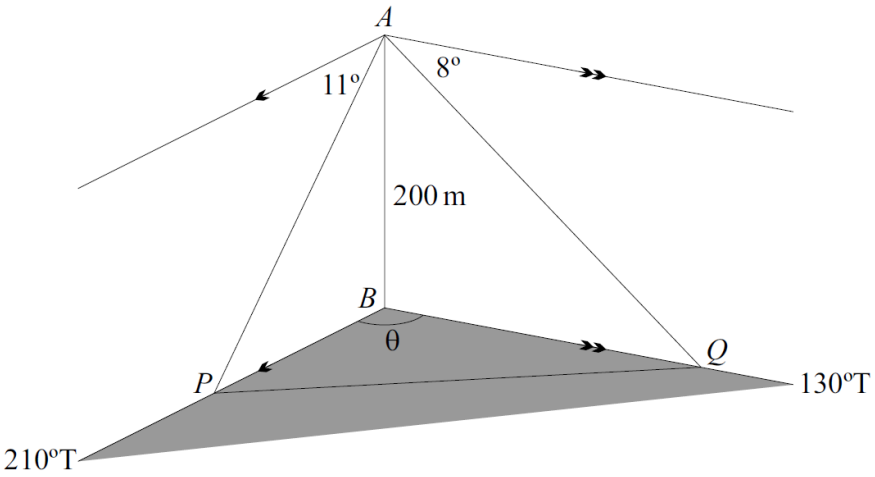
Marks

(c) The diagram above shows a cyclic quadrilateral $ABMN$. Point P lies on AN such that $AB = AP$ and BP produced meets the circle again at D and AM at R. The chord MD intersects AN at Q.



Copy or trace the diagram and show that $QPRM$ is a cyclic quadrilateral. **3**

(d) A jogger is first observed from the top of a vertical 200m cliff, AB at an angle of depression of $8°$ when he is at the point Q. Ten minutes later he is observed at point P with an angle of depression of $11°$. The bearing of Q from B is $130°\text{T}$ and the bearing of P from B is $210°\text{T}$. Let $\angle BPQ = \theta$.



(i) Show that $PQ^2 = 200^2\left(\tan^2 79° + \tan^2 82° - 2\tan 79°\tan 82°\cos\theta\right)$. **2**

(ii) Find the average speed of the jogger in km/h, as he runs in a straight line from P to Q, correct to one decimal place. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

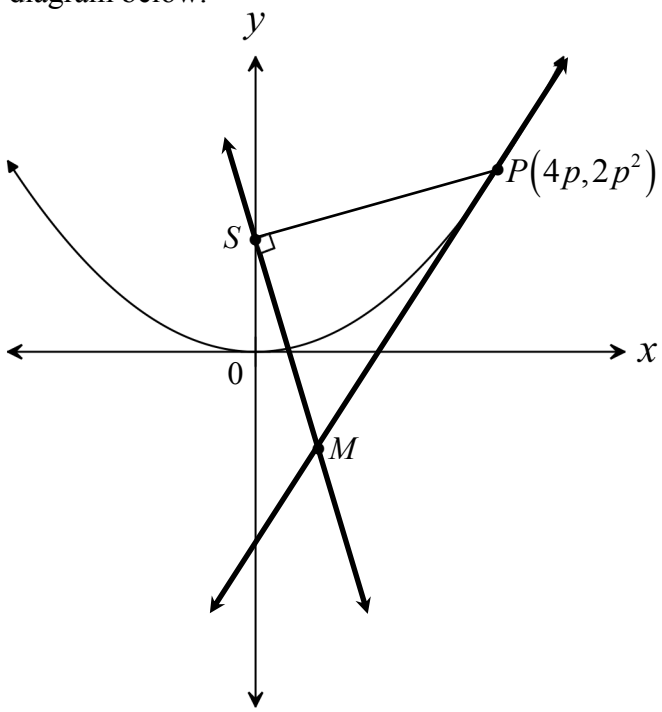
Marks

(a) Solve the following equation for $0° \le \theta \le 180°$:
$3\sin^2\theta + 3\cos^2\theta + 3\tan^2\theta + 3\cot^2\theta + 3\sec^2\theta + 3\operatorname{cosec}^2\theta = 29$. **3**

(b) (i) By considering the difference of two cubes, show that
$$(2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}} = \frac{x}{(2+x)^{\frac{2}{3}} + 2^{\frac{1}{3}}(2+x)^{\frac{1}{3}} + 2^{\frac{2}{3}}}.$$ **2**

(ii) Hence, or otherwise, evaluate $\lim\limits_{x\to 0} \dfrac{(2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{x}$. **2**

(c) $P\left(4p, 2p^2\right)$ is a point on the parabola $x^2 = 8y$, with focus S, as shown in the diagram below.



Not to Scale

(i) Show that the equation of the tangent at P is given by $y = px - 2p^2$. **2**

(ii) Show that the equation of the line through S and perpendicular to SP is $2px + (p^2 - 1)y = 2(p^2 - 1)$. **2**

(iii) The tangent and the line described in part (ii) meet at M.
Prove that the coordinates of M are $\left(\dfrac{2\left(p^2-1\right)}{p}, -2\right)$. **2**

(iv) Show that the area of $\Delta PSM = \dfrac{2\left(p^2+1\right)^2}{|p|}$. **2**

End of Examination