Name:	
Class:	

PRELIMINARY 2016

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 6 - 9, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 85

Section I: 5 marks

- Attempt Question 1 – 5.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 8 minutes for this section.

Section II: 80 Marks

- Attempt Question 6 – 9
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 1 hour & 52 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 6, Question 7, etc. Each question must show your Candidate Number.

To be answered on Multiple Choice Sheet

Question 1

Two straight lines have gradients m-1 and m+1. The acute angle between the lines is θ. Which of the following is an expression for $\tan\theta$?

(A) $\left|\frac{2}{m}\right|$

(B) $\left|\frac{2}{m^2}\right|$

(C) $\left|\frac{2}{2-m^2}\right|$

(D) $\left|\frac{2m}{2-m^2}\right|$

Question 2

P(x) is an odd polynomial. When P(x) is divided by x-2 the remainder is 5. What is the remainder when P(x) is divided by (x+2).

(A) -5

(B) -5x

(C) 5x

(D) 5

Question 3

What ratio does the point P(10,11) divide the interval AB,where A(-2,3) and B(7,9)?

(A) 1:4

(B) 4:-1

(C) 1:-4

(D) 4:1

Question 4

The equation of the normal to the parabola $x^2 = 4ay$ at the variable point P (2ap, ap^2) is given by $x + py = 2ap + ap^3$.

How many different values of p are there such that the normal passes through the focus of the parabola?

(A) 0

(B) 1

(C) 2

(D) 3

Question 5

In how many ways can 5 letters be chosen from the letters ARRANGE?

(A) 9

(B) 10

(C) 12

(D) 21

Question 6 (20 Marks) (Start a NEW PAGE)

(a) Solve the inequality $\frac{3x}{3x-4} \leq 3$ 3

(b) Prove that $\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} = \text{cosec}\,\theta$ 2

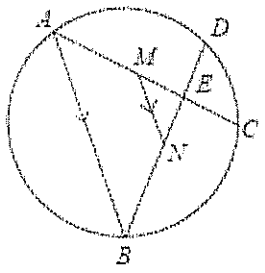
(c) What is the general solution for $2\cos x = \sqrt{3}$ 1

(d) A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four people wish to sit on one particular side and two on the other side. In how many ways can they be seated? 2

(e) (i) In how many ways can the letters of LOUGHBOROUGH be arranged in a line? 2

(ii) If all the O's have to be together how many arrangements are there? 2

(f) AC and BD are two chords of a circle which intersect at E inside the circle. M is the point on AE and N is a point on BD such that $MN \parallel AB$.



(i) Give a reason why $\angle BDC = \angle BAC$ 1

(ii) Hence show that MNCD is a cyclic quadrilateral. 2

(g)

(i) Show that x-2 is a factor of $x^3 - 4x^2 + 3x + 2$ 1

(ii) Factorise fully $P(x) = x^3 - 4x^2 + 3x + 2$ 2

(iii) Sketch P(x) showing all intercepts(do not use calculus) 1

(iv) Hence solve the inequation $x^3 - 4x^2 + 3x + 2 < 0$ 1

Question 7 (20 Marks) (Start a NEW PAGE)

(a) Consider the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where a,b,c and d are real numbers and $a \neq 0$. Let α, β, γ be zeroes of f(x)

(i) Write done an expression for $\alpha + \beta + \gamma$ 1

All cubic polynomials have a single point of inflexion when the second derivative is equal to zero

(ii) Using part (i) or otherwise show that the x-coordinate of the point of inflexion on the curve y = f(x) is given by 2

$$x = \frac{\alpha + \beta + \gamma}{3}$$

(iii) The cubic polynomial has x-intercepts at x =1,-3 and 4. Find the x-coordinate of the point of inflexion of the cubic polynomial. 1

(b) Solve $\cos x - \sqrt{3}\sin x + 1 = 0,\ 0° \leq x \leq 360°$ 3

(c) $P(8p, 4p^2)$ is a point on the parabola $x^2 = 16y$.

The equation of the normal at P is given by $x + py = 4p^3 + 8p$

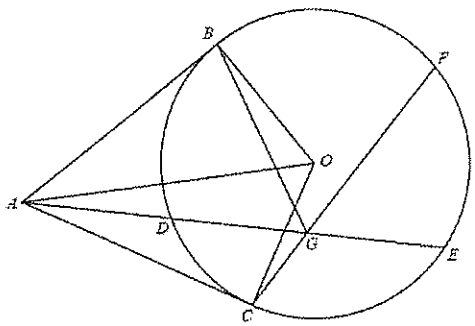
(i) Find the equation of the normal at P 2

(ii) Show the point of intersection R of the normal at P and Q is given by 2

R $(-4pq(p+q), 4(p^2 + pq + q^2) + 8)$

(iii) If pq=2. Find the locus of R 2

(d) In the diagram,AB and AC are tangents from A to the circle centre O, meeting the circle at B and C. AE is a secant of the circle, intersecting it at D and E with G the midpoint of DE. CG produced meets the circle at F. You may assume ABOC is a cyclic quadrilateral



Answer the question on the sheet provided

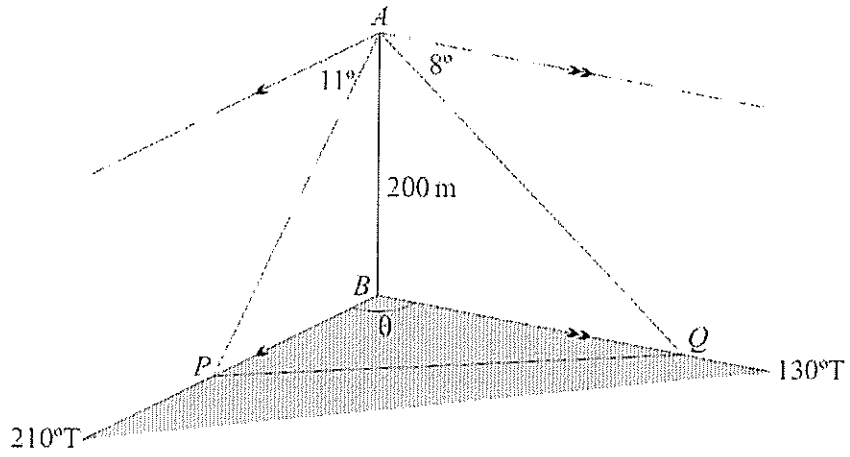
(e) (i) Show that AOGC is a cyclic quadrilateral 3

(ii) Construct BC and BF and let $\angle ABC = \theta$ 4

Prove that BF is parallel to AE

Question 8 (20 Marks) (Start a NEW PAGE)

a)



HMAS Ruse is enroute to carry out its latest mission in Townsville. It is first observed from the top of a 200m cliff, AB at an angle of depression of 8° when it is at the point Q. Ten minutes later it is observed at point P with an angle of depression of 11°. Let $\angle PBQ = \theta$. The bearing of Q from B is $130°T$ and the bearing of P from B is $210°T$.

(i) Show that $PQ^2 = 200^2(\tan^2 79° + \tan^2 82° - 2\tan 79° \tan 82° \cos\theta)$. 2

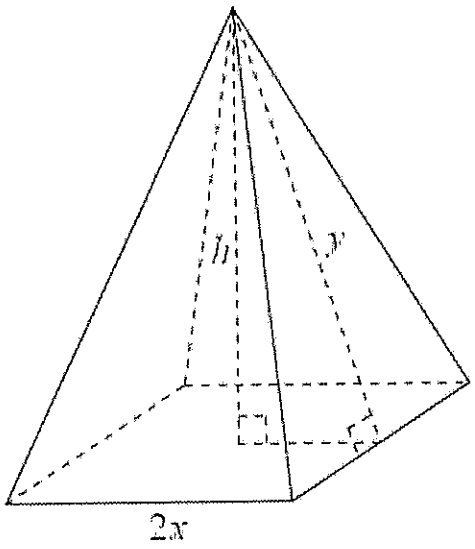
(ii) Find the speed of HMAS Ruse in km\h correct to three significant figures 3

(b) A school band is to be formed with a brass section containing 8 students and a percussion section containing 4 students.

(i) In how many ways can the band be formed if 12 students audition for the brass section and 10 students audition for the percussion section? 1

(ii) In how many ways can the band be formed if Paul who is a brass player and Lisa who is a percussion player won't perform in the together. 2

(c)
A closed right pyramid has a square base of side length 2x cm and its triangular faces each have perpendicular height ycm, as shown in the diagram



(i) This pyramid is to be constructed using $400\,cm^2$ of cardboard 2

Show that $y = \frac{100}{x} - x$

(ii) Show that the perpendicular height h of the pyramid is given by 2

$$h^2 = \frac{10000}{x^2} - 200$$

(iii) Show that the volume V of the pyramid is given by 2

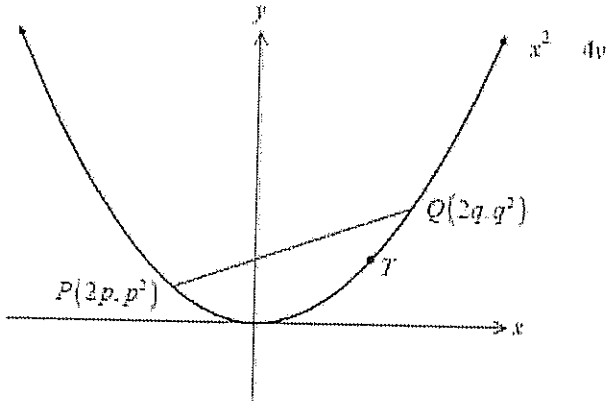
$$V = \frac{40}{3} x\sqrt{100 - 2x^2}$$

(iv) Show that $\frac{dV}{dx} = \frac{160(25 - x^2)}{3\sqrt{100 - 2x^2}}$ 3

(v) Find the base length and perpendicular height of such a pyramid with maximum volume. Justify your answer. 3

Question 9 (20 Marks) (Start a NEW PAGE)

(a) Consider the parabola $x^2 = 4y$ with $P(2p, p^2)$ and $Q(2q, q^2)$



(i) Find the equation of the chord PQ. 2

(ii) Show that if PQ is a focal chord pq=-1 1

(iii) $T(2t, t^2)$, t >0 and $R(2r, r^2)$ are two other points on the parabola distinct from P and Q 3
If TR is also a focal chord and P,T,Q and R are concyclic
Show that $p^2 + q^2 = t^2 + r^2$

(b) One root of the equation $x^3 + ax^2 + bx + c$ is the sum of the other two roots.
Show that $a^3 - 4ab + 8c = 0$ 4

(c) Five women and four men are to be seated at a round table

(i) In how many ways may this be done without restrictions 1

(ii) In how many ways may this be done if no two men are to be seated together 2

(iii) If one man and one woman are a married couple, how many ways can they be seated together, given the conditions of part(ii) 3

(d) (i) Show that $a^2 + b^2 \geq 2ab$ for any values of a and b 2

(ii) Hence show that $\tan^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta \geq \sin\theta + \sec\theta + \cot\theta$ for all values of θ 2

END OF PAPER