Student Name: _____

Teacher: _____

2023

Hurlstone Agricultural High School
Year 11 Assessment Task 3

Mathematics Extension 1

Examiners

- **Mr G Huxley**
- **Ms T Tarannum**
- **Mr G Rawson**
- **Mrs P Biczo**
- **Mr B Hoang**

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use
- In Questions 11 to 15, show relevant mathematical reasoning and/or calculations

Total marks: 60

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II – 50 marks (pages 5–8)

- Attempt Questions 11 - 15
- Allow about 75 minutes for this section.
- For each question use a separate answer booklet.

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

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1. In how many ways can all the letters of the word **EPSILON** be arranged such that the three vowels are all next to each other?

A. 7! **B.** 5!

C. 4!5! **D.** 3!5!

2. A car hire company has 10 sports cars and 16 four-wheel drive cars.
The company is to make a leaflet for a special offer.
This leaflet is to contain 4 sports cars and 6 four-wheel drive cars.
In how many different ways could the cars in the leaflet be chosen?

A. ${}^{10}P_4 \times {}^{16}P_6$ **B.** ${}^{10}C_4 \times {}^{16}C_6$

C. ${}^{10}P_4 + {}^{16}P_6$ **D.** ${}^{10}C_4 + {}^{16}C_6$

3. Which of the following is equivalent to $\tan\left(\frac{\pi}{4} + x\right)$?

A. $\frac{\sec^2 x}{1-\tan^2 x}$ **B.** $\frac{\cos x - \sin x}{\cos x + \sin x}$

C. $\frac{\sin x + \cos x}{\sin x - \cos x}$ **D.** $\frac{\cos x + \sin x}{\cos x - \sin x}$

4. It is given that $\cos\theta = \frac{3}{5}$ for $\frac{3\pi}{2} \leq \theta \leq 2\pi$. What is the value of $\sin 2\theta$ equivalent to?

A. $-\frac{24}{25}$ **B.** $\frac{24}{25}$

C. $-\frac{4}{5}$ **D.** $\frac{4}{5}$

5. How many solutions are there to the equation $3cos^2x = 2$ when $0 \leq x \leq 4\pi$?

A. 2 **B.** 4

C. 8 **D.** 12

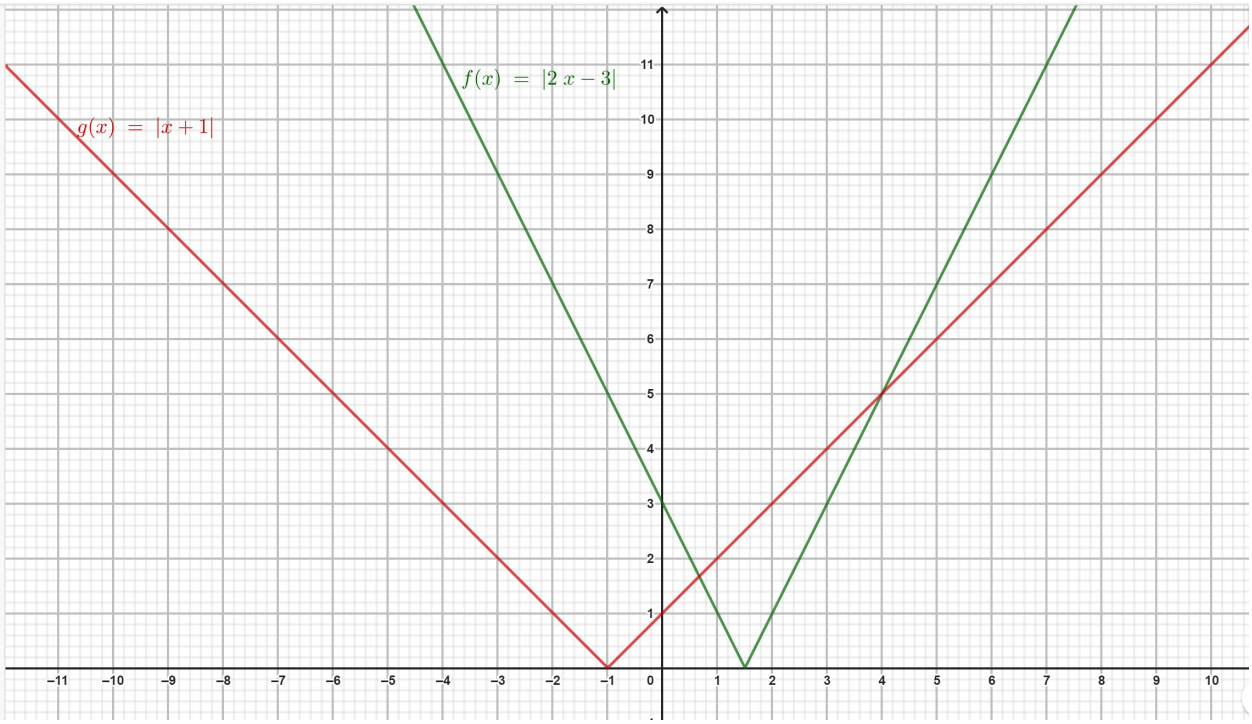
6. The *t* results are to be used to solve the equation $5 \sin x - 2 \cos x = 2$, for *x* values such that $0° \leq x \leq 360°$.

Which value of *x* needs to be considered because it won't be "captured" using this method?

A. $x = 90°$ **B.** $x = 180°$

C. $x = 0°$ **D.** $x = 270°$

7. The graph of $f(x) = |2x - 3|$ and $g(x) = |x + 1|$ are shown below.



Which of the following is the solution to the inequality $|2x - 3| + |x + 1| \leq 4$?

A. $[0,2]$ **B.** $(-\infty, 0] \cup [2, \infty)$

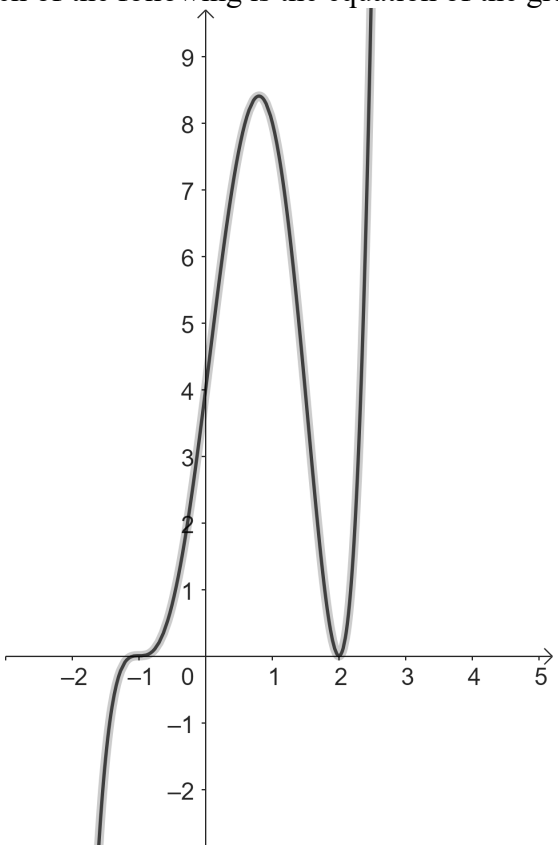
C. $[2, \infty)$ **D.** $(-\infty, 0]$

8. A curve is described by the parametric equation below.
$x = \frac{t}{2}$ and $y = 3t^2$
What is the cartesian equation of the curve?

A. $x = 12y^2$ **B.** $y = \frac{3x^2}{4}$

C. $y = \frac{2}{x}$ **D.** $y = 12x^2$

9. Which of the following is the equation of the graph shown?



A. $y = (x+1)(x-2)^2$ **B.** $y = (x-1)^3(x+2)^2$

C. $y = (x-1)^3(2-x)^2$ **D.** $y = (x+1)^3(x-2)^2$

10. When $P(x) = x^3 + 2x^2 - 3x + k$ is divided by $x + 2$, the remainder is -4.
Which of the following is value of k?

A. $k = -10$ **B.** $k = 2$

C. $k = 6$ **D.** $k = 10$

~ END OF SECTION I ~

Section II

50 marks
Attempt Questions 11 to 15
Allow about 75 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (10 marks) **Marks**

(a) Express $\frac{^{10}C_k}{^{9}C_k}$ in simplest form **2**

(b) A debating group consists of 12 students, 8 of whom are girls.
How many debating teams of three students can be formed if:

(i) there are no girls at all? **1**

(ii) there is exactly one girl? **1**

(iii) there is at least one girl? **2**

(c) At a fund-raising dinner, each rectangular table has 9 seats,
5 facing the stage and 4 with their backs to the stage.

In how many ways can nine people be seated at a particular table if:

(i) Lewis and Huey must sit on the same side? **2**

(ii) Lewis and Huey must sit on opposite sides? **2**

~ End of question 11 ~

Question 12 (10 marks) **Marks**

(a) Find the value of $\cos 105°$ in simplest exact form.

(Your response is allowed to include an irrational denominator) **2**

(b) With the use of *t*- results, show that: $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$ **3**

(c) What is the maximum value of the expression $6 \sin x \cos x$?

Justify your answer by using a trigonometric identity. **2**

(d) It is given that $\cos 3\theta = \cos^3 \theta - 3 \sin^2 \theta \cos \theta$. (You DO NOT need to prove this)

(i) By considering $\sin 3\theta = \sin(2\theta + \theta)$, find a similar expression for $\sin 3\theta$. **2**

(ii) Hence or otherwise, show that $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = 2$ **1**

~ End of question 12 ~

Question 13 (10 marks) **Marks**

(a) Let $R\ sin\alpha = 3$
$R\ cos\alpha = -4$ where $R > 0$,

What is the size of angle α, correct to the nearest minute? **1**

(b) (i) Express $\sqrt{3} \sin x + \cos x$ in the form $R \sin(x + \alpha)$. **2**

(ii) Hence or otherwise, solve for $0 \leq x \leq 2\pi$:

$\sqrt{3} \sin x + \cos x = -1$. **2**

(c) Solve, correct to the nearest degree:

$\sin \theta + 6 \cos^2 \theta = 5$ where $0° \leq \theta \leq 360°$. **3**

(d) Solve, correct to the nearest degree:

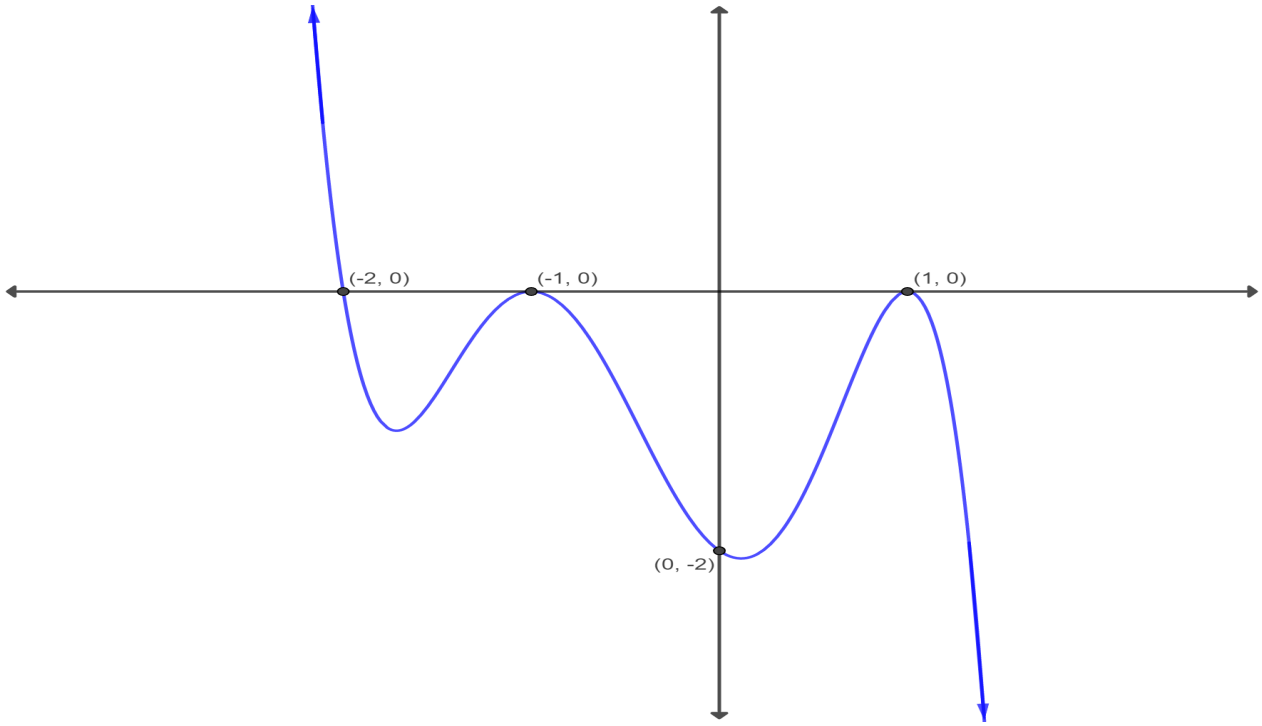
$\cos 3x = \frac{4}{7}$ where $0° \leq x \leq 360°$ **2**

~ End of question 13 ~

Question 14 (10 marks) **Marks**

(a) Solve $\frac{1}{3x-1} \leq 5$ **3**

(b) The diagram below shows the graph of a function $y = f(x)$.
The function has zeroes at $x = -2$, $x = -1$ and $x = 1$, and a y-intercept at -2 as shown.



In your answer booklet, the sketch of the graph of $y = f(x)$ is provided.

Sketch $y = \frac{1}{f(x)}$ on the same axes.
Show any asymptotes and intercepts, together with the location of the points corresponding to the labelled points on $y = f(x)$. **3**

(c) A function f is defined by the parametric equations

$$x = 4 - 2t \text{ and } y = 3 + 6t - 4t^2$$

(i) Find the cartesian equation for the function f. **1**

(ii) Hence or otherwise sketch the graph of the function f.

Show the turning point and the y-intercept on the graph. **2**

(iii) What is the value of the parameter t at the turning point? **1**

~ End of question 14 ~

Question 15 (10 marks) **Marks**

(a) (i) Use long division, to divide $P(x) = 6x^3 + 5x^2 - 33x + 20$ by $2x - 3$. **2**

(ii) Hence write $P(x)$ in the form $P(x) = (2x - 3)Q(x) + R(x)$. **1**

(b) If α, β, γ are the roots of the equation $x^3 - 3x^2 + 3x - 7 = 0$, find the value of $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$. **2**

(c) (i) Factorise $P(x) = x^3 - 4x^2 - 3x + 18$. **2**

(ii) Hence solve $(1 - x)^3 - 4(1 - x)^2 - 3(1 - x) + 18 = 0$. **1**

(d) It is given that $x = \alpha$ is a zero of multiplicity 2 of the polynomial $P(x)$, hence $P(x) = (x - \alpha)^2 Q(x)$ where $Q(x)$ is a polynomial.

Prove that $x = \alpha$ is a zero of multiplicity 1 of $P'(x)$. **2**

~ End of question 15 ~

~ END OF SECTION II ~

~ END OF PAPER ~

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

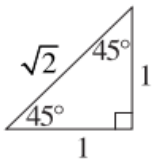
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

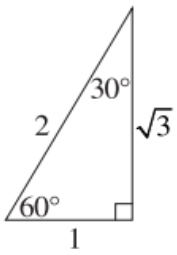
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

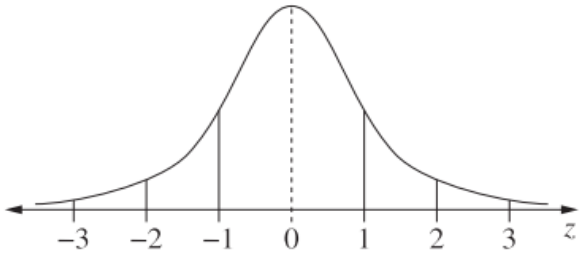
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Student Name: _____

Teacher: _____

2023 Year 11 Mathematics Extension 1
Assessment Task 3

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Start Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

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