FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS
Extension
PRELIMINARY ASSESSMENT TASK 3
ANSWERS COVER SHEET

Name:_____

QUESTION	MARK	PE2	PE3	PE4	PE5	PE6
Q1 - Q5	/5		✓			✓
Q6	/14		✓			✓
Q7	/13		✓			✓
Q8	/15		✓			✓
Q9	/16		✓			✓
Q10	/8		✓			✓
TOTAL						
	/71		/71			/71

Preliminary Learning Outcome. Mathematics Extension 1

PE2 uses multi-step deductive reasoning in a variety of contexts.

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations.

PE4 uses parametric representation together with differentiation to identify geometric properties of parabolas.

PE5 determines derivatives which require the application of more than one rule of differentiation.

PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of forms

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

Task 3

Year 12 Mathematics Extension 1 2016

Time Allowed: 50 minutes

Instructions:

- There are 10 questions in this paper. All questions are compulsory.
- Use blue or black pen.
- For Questions 1 – 5, write the letter corresponding to the correct answer on your answer paper.
- Start each question (6 - 10) on a new sheet of paper.
- Write on one side of the paper only.
- Show all necessary working.
- Board-approved calculators may be used.
- Mathematics Reference sheets are provided.
- Marks may be deducted for careless or badly arranged work.
- Write 'End of Solutions' at the conclusion of your solutions to the task.

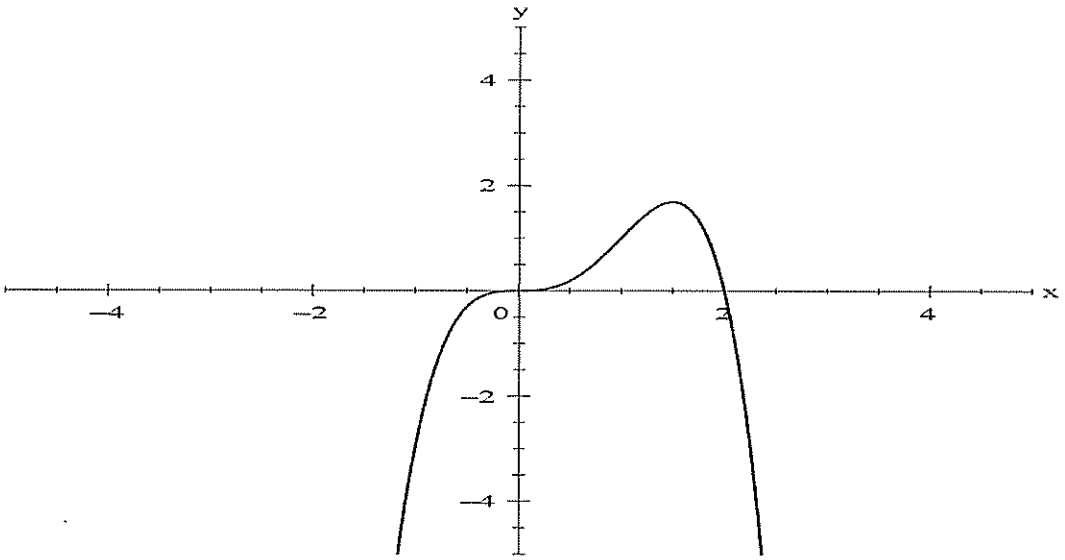
Questions 1 – 5 (5 marks)

Write the letter corresponding to the correct answer on your answer sheet.

1. When the polynomial $P(x) = x^3 - 5x^2 + kx + 2$ is divided by $(x + 1)$, the remainder is 3. What is the value of k?

   A. 5 B. – 5 C. – 7 D. 7

2. The graph of $y = P(x)$ is shown below.



   Which of the following could be the polynomial $y = P(x)$?

   A. $P(x) = x^3(2 - x)$ B. $P(x) = x^2(2 - x)^2$

   C. $P(x) = x^3(x - 2)$ D. $P(x) = -x^3(x + 2)$

3. For the polynomial equation $6 - 4x + 10x^2 - 8x^3 = 0$, the sum of the roots divided by the product of the roots would be:

   A. $\frac{5}{3}$ B. $\frac{-4}{3}$ C. $\frac{-1}{2}$ D. $\frac{5}{4}$

4. The remainder of the division $\frac{x^5 + 1}{x^2 - 1}$ is equal to

   A. 1 B. 2 C. $x + 2$ D. $x + 1$

5. Which group of three numbers could be the roots of the polynomial $x^3 + ax^2 - 41x + 42 = 0$?

   A. 2, 3, 7 B. 1, - 6, 7

   C. - 1, - 2, 21 D. - 1, - 3, - 14

Questions 6 – 10

- *Start each question on a new sheet of paper.*
- *Write on one side of the paper only.*
- *Show all necessary working.*

Question 6 (14 marks)

a. State whether the following are polynomials or not. If not, state why.

(i) $\frac{1}{x} + 2x^{-3}$ [2]

(ii) $2x^3 - 3x + 7$ [2]

(iii) $\sqrt{3}\,x - 4$ [2]

b. Given that $A(x) = 3x^2 - 2x + 1$

$B(x) = 5x - 2$

$C(x) = 2x^4 - 5x^2 + 3x + 4$

Simplify: (i) $A(x) + C(x)$ [2]

(ii) $A(x) \times B(x)$ [3]

(iii) $A(x) - 3C(x) + 2B(x)$ [3]

Question 7 (13 marks)

a. Perform the following long divisions, expressing your answer in the form $P(x) = D(x) \times Q(x) + R(x)$.

(i) $(x^4 - 3x^2 + 2) \div (x - 1)$ [3]

(ii) $(4x^3 - 8x^2 + 8x - 1) \div (2x + 1)$ [3]

b. Without using long division, find the remainder when

(i) $x^3 + 3x^2 + 2x - 7$ is divided by $(x + 2)$ [2]

(ii) $x^4 - 3x^3 + x + 1$ is divided by $(2x - 1)$ [2]

c. When $P(x) = x^3 + x^2 + ax + a^2$ is divided by $(x - 2)$, the remainder is 27.

(i) Show that $a^2 + 2a - 15 = 0$ [2]

(ii) Hence, find the value of a. [1]

Question 8 (15 marks)

a. Given that $P(x) = 2x^3 - x^2 - 7x + 6$

(i) Show that $(x - 1)$ is a factor of $P(x)$. [1]

(ii) Express $P(x)$ as a product of its linear factors. [3]

(iii) Sketch the graph of $y = P(x)$. [3]

(iv) Find the values of x for which $P(x) > 0$ [2]

b. Sketch the following, clearly showing the intersection with the axes.

(i) $f(x) = x(x - 2)^2(x + 3)$ [3]

(ii) $f(x) = x(2x - 1)(x - 3)^3$ [3]

Question 9 (16 marks)

a. If α, β and γ are the roots of $P(x) = 2x^3 - 5x^2 - 3x + 1$, find the values of:

(i) $\alpha + \beta + \gamma$ [1]

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ [1]

(iii) $\alpha\beta\gamma$ [1]

(iv) $3\alpha + 3\beta + 3\gamma - 4\alpha\beta\gamma$ [2]

(v) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ [2]

(vi) $\alpha^2 + \beta^2 + \gamma^2$ [3]

b. (i) Verify that
$(\alpha\beta + \beta\gamma + \alpha\gamma)^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. [2]

(ii) Hence, or otherwise, if α, β and γ are the roots of
$x^3 - 3x^2 - x + 3 = 0$ evaluate $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. [4]

Question 10 (8 marks)

a. A polynomial is given by $P(x) = x^3 + ax + b$ for some real numbers a and b.
$(x - 2)$ is a factor of $P(x)$ and when $P(x)$ is divided by x the remainder is 2.
Find the values of a and b. [3]

b. The equation $x^3 - kx - 2 = 0$ has two equal roots. Use the relationship between the roots and the coefficients to find the roots and the value of k. [5]

End of Examination