2022 YEAR 11 EXAMINATION

Mathematics Advanced

Monday 12th September

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

Name:_____

Student Number:_____

A student:

MA11-1	uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
MA11-2	uses the concepts of functions and relations to model, analyse and solve practical problems
MA11-3	uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes
MA11-4	uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
MA11-5	interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems
MA11-6	manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems
MA11-7	uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions
MA11-9	provides reasoning to support conclusions which are appropriate to the context

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1 Which of the following statements is equivalent to $\tan^2 x$?

A. $-\sec^2 x - 1$

B. $1 - \sec^2 x$

C. $1 + \sec^2 x$

D. $\sec^2 x - 1$

2 What is the solution of the equation $2\cos x + 1 = 0$ for $0° \le x \le 360°$?

A. $x = 120°$ or $x = 240°$

B. $x = 150°$ or $x = 210°$

C. $x = 120°$ or $x = 300°$

D. $x = 150°$ or $x = 330°$

3 Which of the following expressions is equal to $\dfrac{x^2 - y^2}{x^2 y - xy^2}$?

A. $\dfrac{1}{xy}$

B. $\dfrac{x + y}{xy}$

C. $\dfrac{x + y}{x - y}$

D. $\dfrac{1}{x} - \dfrac{1}{y}$

4 What is the radius of the circle $x(x-4)+y(y-2)=4$?

A. 1

B. 2

C. 3

D. 4

5 If $y=2x\sqrt{x}$ what is an expression for $\frac{dy}{dx}$?

A. $\frac{dy}{dx}=\sqrt{x}$

B. $\frac{dy}{dx}=2\sqrt{x}$

C. $\frac{dy}{dx}=3\sqrt{x}$

D. $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$

6 What is the value of $\log_2\frac{1}{4}+\log_2\sqrt{2}$?

A. $-\frac{5}{2}$

B. $-\frac{3}{2}$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

7 What is the range of the function $f(x) = 2\sin x - 1$?

A. $[-3,1]$

B. $[-2,2]$

C. $[-1,1]$

D. $[-1,3]$

8 A particle is moving in a straight line. At time t seconds it has displacement $x = 4(t-1)^4$ metres from a fixed point O on the line and velocity v m/s. What is the time taken for the speed of the particle to equal 2 m/s ?

A. $\frac{1}{2}$ second

B. 1 second

C. $\frac{3}{2}$ seconds

D. 2 seconds

9 If $y = \frac{e^x}{e^x + 1}$ which of the following is correct?

A. $x = \ln\left(\frac{1-y}{y}\right)$

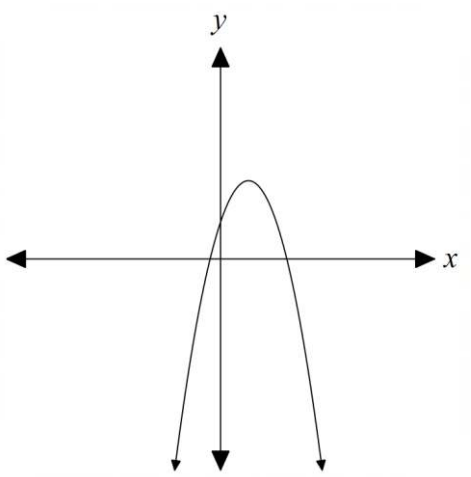
B. $x = \ln\left(\frac{y}{1-y}\right)$

C. $x = \ln\left(\frac{y}{1+y}\right)$

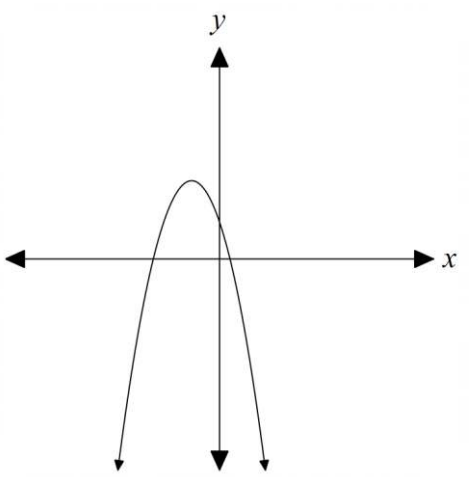
D. $x = \ln\left(\frac{1+y}{y}\right)$

10 Which of the following could be the graph of $y = -ax^2 + bx + c$ given that a, b and c are positive constants?

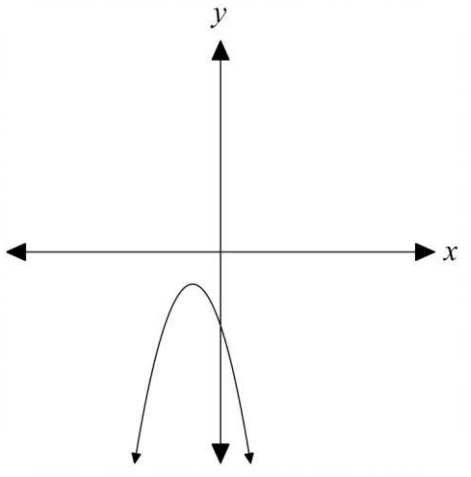
A.



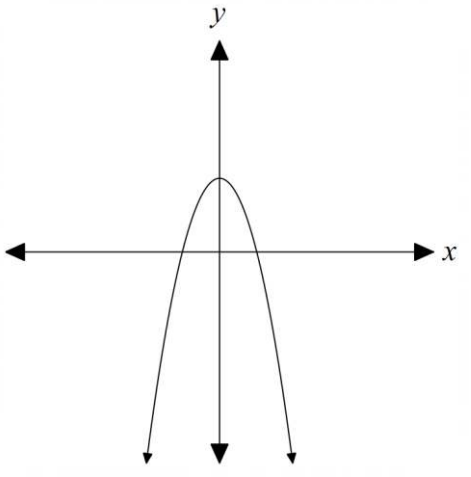
B.



C.



D.



Student Number: _____

2022 YEAR 11 EXAMINATION

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11 - 35
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is provided at the back of each booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Simplify the expression $\left(\frac{2x^2}{y^3}\right)^{-2} \div \left(4x^2y\right)^3$. **2**

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Question 12 (2 mark)

Given $\tan\theta = -\frac{8}{5}$ for an obtuse angle θ, find the exact value of $\sin\theta$. **2**

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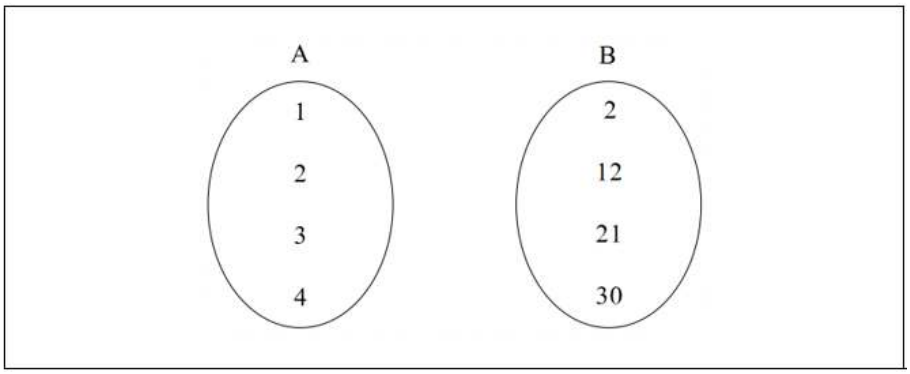
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Question 13 (1 mark)

Draw arrows connecting numbers in Set A to numbers in Set B which demonstrate a many-to-many relation f from A to B. **1**



Question 14 (2 marks)

Find the gradient of the curve $y = \frac{1}{x^2}$ at the point $\left(\frac{1}{2}, 4\right)$. **2**

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Question 15 (2 marks)

Given $\log_a 2 = x$ and $\log_a 6 = y$, find an expression for $\log_a 24$ in terms of x and y. **2**

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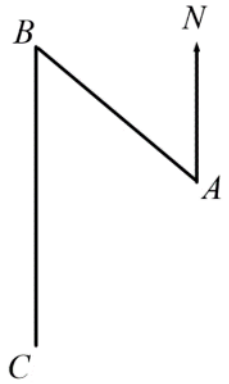
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Question 16 (6 marks)

A plane flies 165 km from point *A* to point *B* on a bearing of 295°.
The plane then flies 220 km due south to point *C* before returning to point *A*.



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(a) Complete the diagram with the information provided and find $\angle ABC$. **2**

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(b) Find the distance of the plane's return trip from point *C* to point *A* to the nearest kilometre. **2**

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(c) Find the bearing of the plane's return trip from point *C* to point *A* to the nearest degree. **2**

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Question 17 (3 marks)

Solve the equations below. Give your answers in exact form.

(a) $2^x = 5$ **1**

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(b) $\left|\log_e(x) - 1\right| = 1.$ **2**

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Questions 11 – 17 are worth 18 marks in total

Section II (Questions 11 – 17) extra writing space.

If you use this space, clearly indicate which question you are answering.

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Section II Answer Booklet 2 **Student Number: _____**

Question 18 (6 marks)

Consider $y = (x+2)^2$.

(a) Find the equation of the normal at the point $(-1, 1)$. **3**

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(b) Find the point A where the normal cuts the x-axis and the point B where the normal cuts the y-axis **2**

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(c) If O is the origin, find the area of ΔAOB. **1**

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Question 19 (4 marks)

(a) Simplify $\sin(-\theta)\cos(90-\theta)$. **1**

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(b) Solve $2\sin^2\theta - 5\sin\theta - 3 = 0$ for $\theta \in [0°, 360°]$. **3**

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Question 20 (6 marks)

A discrete probability distribution for random variable X is given in the table below. X can take integer values from 1 to 5, a and b are positive constants such that $0 \le a \le 1$ and $0 \le b \le 1$.

X	1	2	3	4	5
$P(X = x)$	a	b	b	b	b

(a) Find the value of a and b, given $E(X) = 1.8$. **4**

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(b) Hence, find Var(X). **2**

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Questions 18 – 20 are worth 16 marks in total

Section II (Questions 18 – 20) extra writing space.

If you use this space, clearly indicate which question you are answering.

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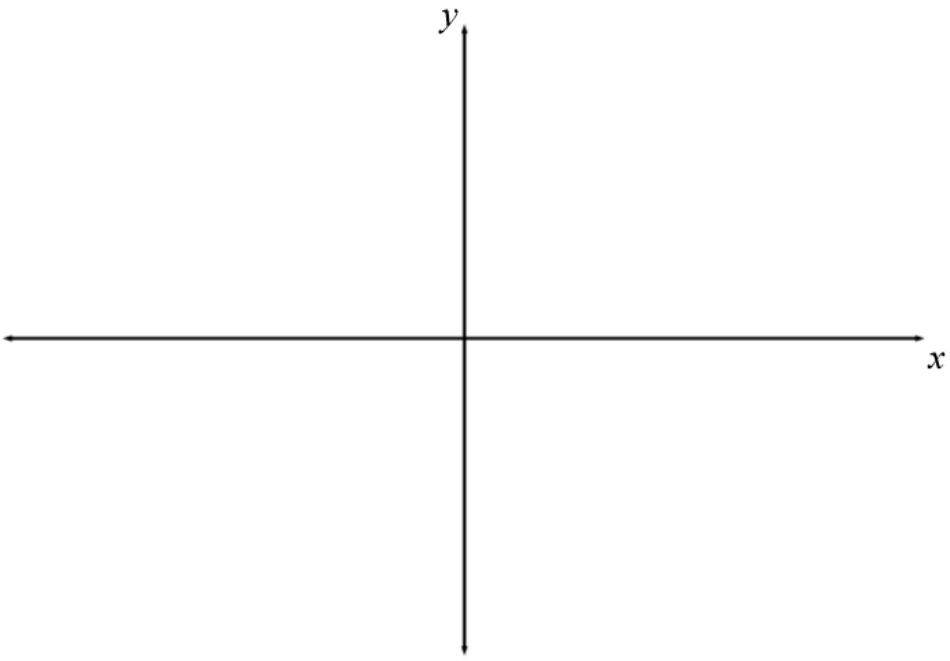
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Section II Answer Booklet 3 **Student Number: _____**

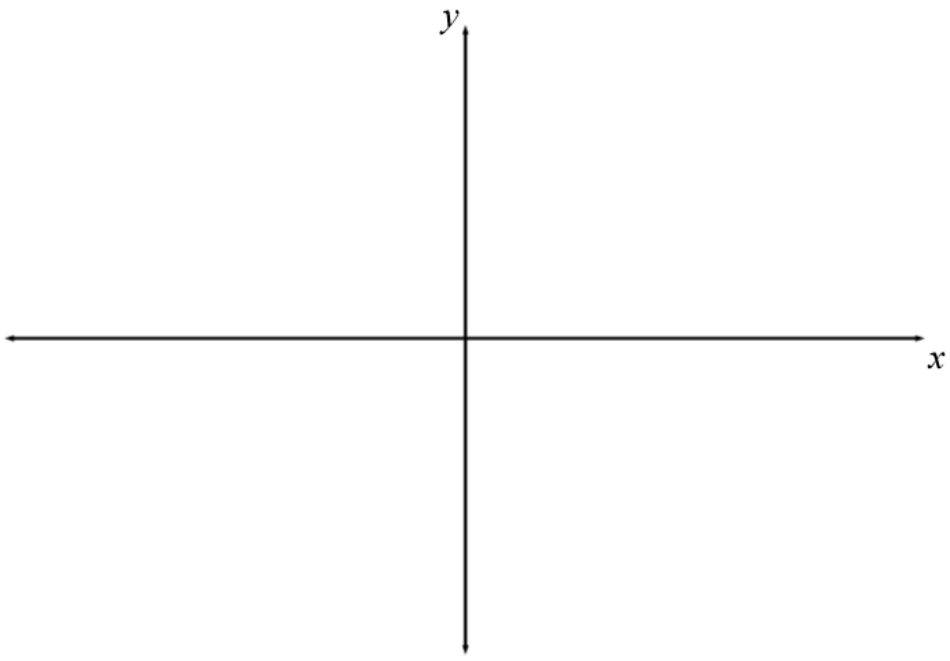
Question 21 (3 marks)

Draw a neat sketch of $y = (x+1)^3 - 2$ on the set of axes provided below, showing x and y intercepts. **3**



Question 22 (2 marks)

Draw a neat sketch of the function $f(x) = e^x + 1$ showing any intercepts on the axes and the equations of any asymptotes. **2**



Question 23 (4 marks)

(a) If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$, find the composition function $f\left(g(x)\right)$. **1**

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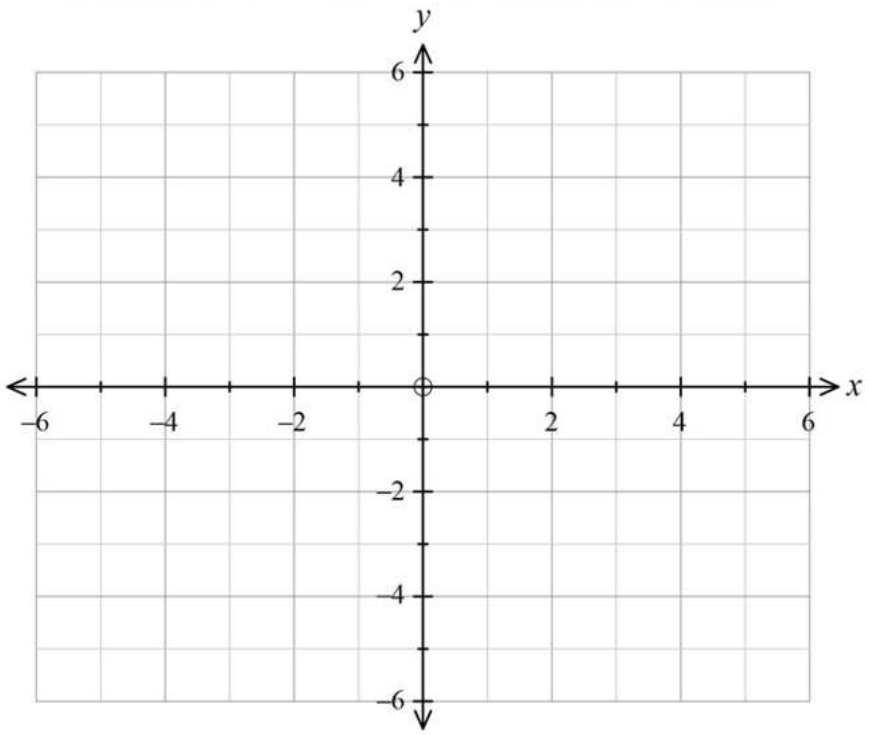
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(b) State the domain of $f\left(g(x)\right)$. **1**

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(c) Sketch the graph $y = f\left(g(x)\right)$ showing any intercepts on the axes. **2**



Question 24 (5 marks)

Differentiate the following.

(a) $y = 2x^7 - 3x + 1$ **1**

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(b) $y = \dfrac{7}{\sqrt{x^2 - 1}}$ **2**

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(c) $y = 4xe^{-x}$ **2**

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Question 25 (2 marks)

(a) Show that the table below represents a discrete probability distribution. **1**

x	0	1	2	3	4
P (X = x)	0.1	0.4	0.3	0.15	0.05

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(b) Hence, find $P(1 \leq x \leq 3)$. **1**

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Question 26 (2 marks)

(a) A coin is tossed twice. Complete the probability distribution table below for the number of heads. **1**

x	0	1	2
$P(X = x)$			

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(b) Hence, find the probability of at least 1 head. **1**

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Questions 21 – 26 are worth 18 marks in total

Section II (Questions 21 – 26) extra writing space.

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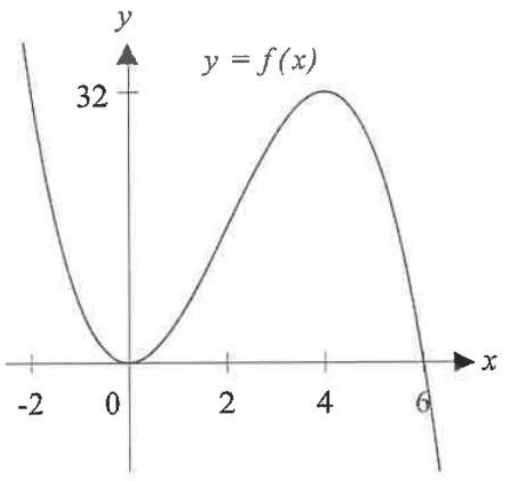
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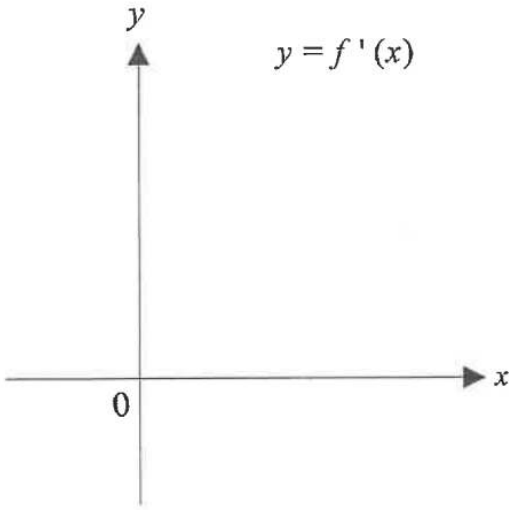
Section II Answer Booklet 4 **Student Number: _____**

Question 27 (2 marks)

The diagram below shows the graph of the curve $y = f(x)$. **2**

Sketch the graph of $y = f'(x)$ showing the shape of the curve and the x intercepts.





Question 28 (3 marks)

Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 + x$. **3**

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Question 29 (5 marks)

(a) Given $y = \dfrac{2x+1}{3x^2+6}$, show that $\dfrac{dy}{dx} = \dfrac{6\left(2-x-x^2\right)}{\left(3x^2+6\right)^2}$. **3**

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(b) Hence, find the values of x where the tangent to the curve $y = \dfrac{2x+1}{3x^2+6}$ is horizontal. **2**

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Question 30 (6 marks)

A city's population is experiencing exponential growth such that the population P at time t in years can be given in by $P = 50000e^{at}$ where a is a positive constant.

(a) What is the initial population of the city? **1**

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(b) If the population is 52400 after 1 year, find the value of a to 4 decimal places. **2**

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(c) Find the average rate of growth of the population in the first 5 years, correct to 1 decimal place. **3**

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Question 31 (4 marks)

(a) Solve $2\sin\theta = -\sqrt{3}$ for $[-180°, 180°]$. **2**

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(b) Solve $\tan^2\theta = 1$ for $0° \le \theta \le 360°$. **2**

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Questions 27 – 31 are worth 20 marks in total

Section II (Questions 27 – 31) extra writing space.

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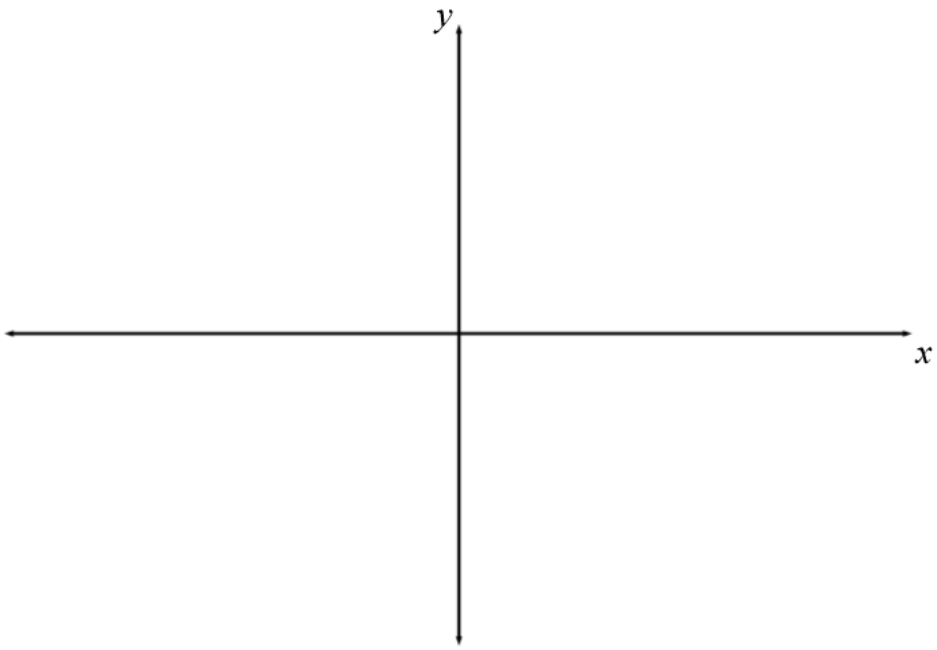
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Section II Answer Booklet 5 **Student Number: _____**

Question 32 (5 marks)

(a) Sketch the graph of $y = \ln(x+1) - 2$, labelling the x-intercept, y-intercept and vertical asymptote. **3**



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(b) Sketch $y = x - 4$ on the number plane above. **2**

How many solutions are there to $x - 4 = \ln(x+1) - 2$?

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Question 33 (5 marks)

(a) Show that $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=2\sec\theta.$ **3**

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(b) Hence, solve $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=2$ for $0°\le\theta\le360°.$ **2**

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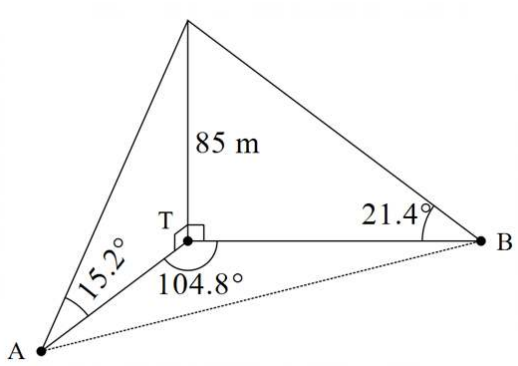
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Question 34 (3 marks)

A radio tower at point T is 85 metres tall and stands perpendicular to the ground. **3**



Angela is at point A where the angle of elevation to the top of the tower is $15.2°$. Beth is at point B where the angle of the elevation to the top of the tower is $21.4°$.
The angle $\angle ATB$ is 104.8°.

The tower is due for repairs and a construction site will be fenced off in the shape of triangle ATB. Find the area of triangle ATB the nearest square metre.

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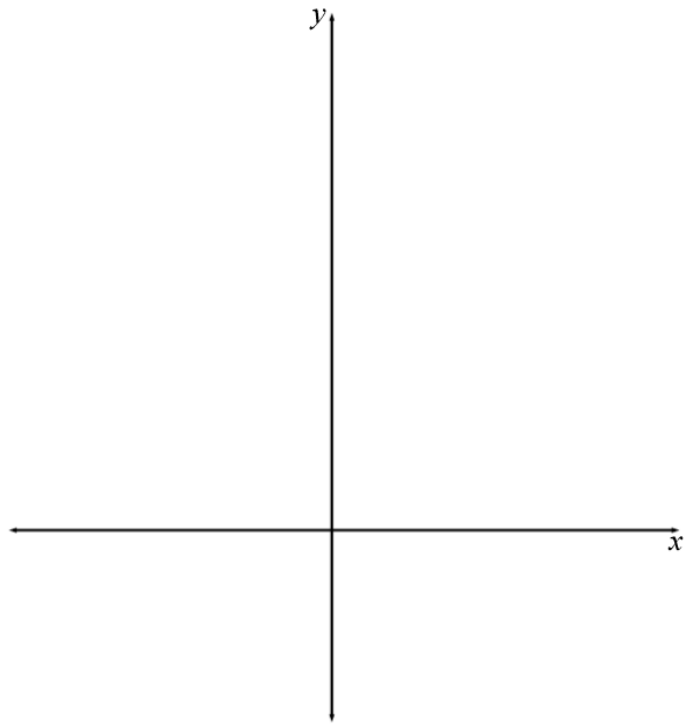
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Question 35 (5 marks)

(a) Draw a neat sketch of $y = x^2 - 5x + 14$ labelling the vertex and any axes intercepts. **2**



(b) The line $y = x + 5$ is a tangent to the curve $y = x^2 - 5x + 14$. **3**
Find the coordinates of the point where the tangent touches the curve and draw your tangent on your graph in part (a).

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Questions 32 – 35 are worth 18 marks in total

End of paper

Section II (Questions 32 – 35) extra writing space.

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STUDENT NUMBER:_____

YEAR 11 MATHEMATICS ADVANCED
2022 EXAMINATION

Answer Sheet for Section I

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭