Name:	
Class:	

YEAR 11
PRELIMINARY EXAMINATION 2012

MATHEMATICS

Time Allowed – 90 minutes
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1 (15 Marks) **Marks**

(a) In the diagram the straight line $3x+2y=0$, the parabola with vertex $V(-4,0)$, its y-intercepts $(0,2)$ and $(0,-2)$ are shown.

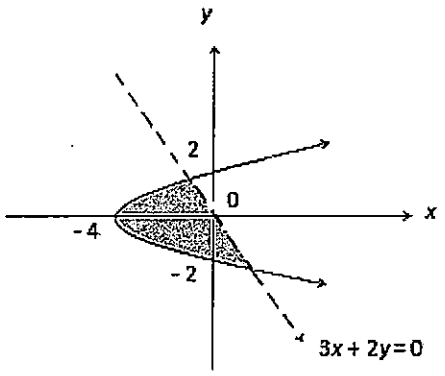


Diagram not to scale

(i) State the equation of the parabola. **1**

(ii) Write down the inequations which describe the shaded region **1**

(b) Solve for x: $\left|\frac{2}{3}x-4\right|=x-1$ **2**

(c) Solve for x: $\log_3(2x-1)+\log_3(x-4)=2$ **3**

(d) The straight line $3x-4y+p=0$ is 2 units from the point $(2,5)$. Find the possible values of p. **3**

(e) Find the values of k for which the quadratic expression: $(k+8)x^2-6x+k$ is positive definite. **3**

(f) Simplify $\left(1-\cos^2\theta\right)\left(\frac{1+\tan^2\theta}{\tan^2\theta}\right)$ **2**

Question 2 (15 marks) (Start A New Page)

(a) Find $\frac{dy}{dx}$ in its simplest form: **5**

(i) $y=e^{5-3x}$

(ii) $y=\left(4\sqrt{x}+3\right)^5$

(iii) $y=\ln(\sin 4x)$

Question 2 continued over the page

Marks

(b) For the quadratic equation $5+3x-2x^2=0$, with roots of α and β, find 4

(i) $\alpha+\beta$ (ii) $\alpha\beta$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

(c) A ship, S, is steering a straight course towards an oil rig, R, on a bearing 220° T. At the same time, a lighthouse, L, is sighted in a direction 290° T. From the charts, it is known that the distance from the rig to the lighthouse is 15 km and the bearing of the rig from the lighthouse is 191° T.

(i) Draw a neat diagram showing the above information. 1
(ii) Find the distance of the ship from the rig to two decimal places. 3

(d) Explain why the curve $y=\sqrt{\tan 3x}$ is rising for all values of x, with $0<x<\frac{\pi}{6}$. in the domain. 2

Question 3 (15 marks) (Start A New Page)

(a) Given that $x=-\frac{1}{2}$ is one root of the following quadratic equation $mx^2-20x+m=0$, find the exact value of the other root. 2

(b) P and Q are the midpoints of the sides AB and AC of ΔABC respectively. PQ is produced to X so that $PQ=QX$. Draw a diagram and prove that:

(i) CX is equal and parallel to BP. 1 3

(ii) $PQ=\frac{1}{2}BC$ 3 1

(c) A point $P(x,y)$ moves so that its distance from the y axis is equal to its distance from the point $(4,2)$.

(i) Show that the locus of P has equation $(y-2)^2=8(x-2)$. 2
(ii) State the focal length, equation of directrix, coordinates of focus and vertex. 3

(d) A garden bed is in the shape of a circle with a minor segment removed as shown. 4

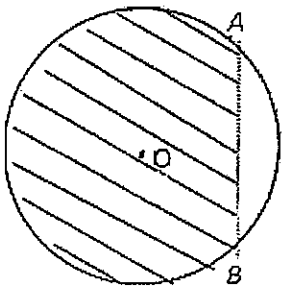


Diagram not to scale

The circle has centre O and radius 5 metres. The length of the straight edge AB is $5\sqrt{3}$ metres.

Find the exact area of the garden bed (shaded region).

Question 4 (15 marks) (Start A New Page) **Marks**

(a) Solve $2\sin^2 x - 3\sin x - 2 = 0$ for $0° \le x \le 360°$. 3

(b) The diagram shows a cylinder inscribed in a sphere. The radius of the sphere is $4\sqrt{6}$ cm. The height of the cylinder is $2h$ cm

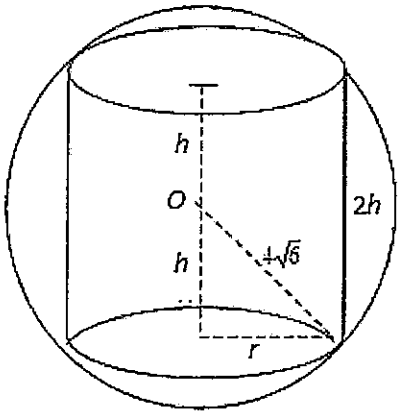


Diagram not to scale

(i) Show that $r = \sqrt{96 - h^2}$ 1

(ii) Hence show that the volume, V, cm^3 of the cylinder is given by: $V = 2\pi(96h - h^3)$. 1

(iii) Find the value of h that will give the cylinder a maximum volume. 3

(iii) Show that the volume of the maximum inscribed cylinder is $\frac{1}{\sqrt{3}}$ of the volume of the sphere. 3

(c) During the 2012 Olympics, in a rifle club, discs were projected along the path of a parabola with equation $y = -\frac{x^2}{25} + 2x$, as shown below.

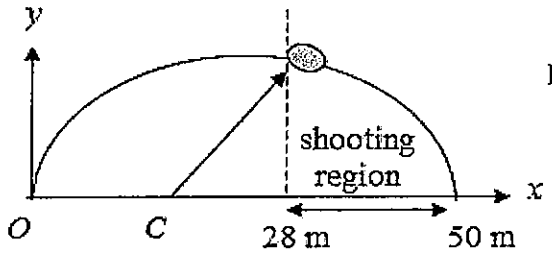


Diagram not to scale

The club enforced some rules which include the following:

- Competitors, C must only shoot at the disc when it is in the shooting region between 28 m and 50 m.
- Competitors must shoot from a position no further than 23 metres from the origin.
- The number of points awarded for each shoot is equal to the height of the disc when first struck by a bullet.
- Bonus points are awarded if the disc is completely destroyed. To guarantee complete destruction, direction of the bullet must be normal to the path of the disc.

Find the position from the origin (nearest metre) and shooting angle (nearest minute) to achieve maximum points. 4

End Of Exam