Student number

Gosford High School

2024 Year 11 Yearly Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- In Questions 11–29, show relevant mathematical reasoning and/ or calculations

Total Marks

80

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Use the *Multiple Choice Answer Sheet* supplied for this section.

Section II – 70 marks

- Attempt Questions 11–29
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt questions 1-10

Use the multiple-choice answer sheet for Question 1-10

1. When $\frac{a-\sqrt{b}}{\sqrt{b}}$ is expressed in the form $c\sqrt{b}-d$:

a) $d=-1$

b) $d=b$

c) $d=1$

d) $d=\frac{a}{b}$

2. A man on a cliff looks down at a ship an angle of depression of 14°. The cliff is 55 metres tall. How far out from the base of the cliff is the ship to the nearest metre?

a) $221m$

b) $14m$

c) $227m$

d) $13m$

3. Given that $\cos\theta=\frac{-4}{7}$ and $\sin\theta>0$, then:

a) $\sin\theta=\frac{3}{7}$

b) $\sec\theta=\frac{7}{4}$

c) $\tan\theta=\frac{-\sqrt{33}}{4}$

d) $\cot\theta=\frac{4}{\sqrt{33}}$

4. Solve $|2x-15|=8$

a) $x=3.5$ or 11.5

b) $x=11.5$ or -11.5

c) $x=23$

d) $x=-3.5$ or 11.5

5. A group of 535 people were surveyed. The people were asked whether they have pets or not. The results are recorded in the table below.

	Have pets	No pets	Total
Male	118	206	324
Females	131	90	221
	249	296	535

What is the approximate probability that a person selected at random would be female or have pets.

a) 63%

b) 41%

c) 24%

d) 47%

6. The probability distribution of a discrete random variable, X, is given by the table below.

x	2	3	4	5	6
$P(X = x)$	0.1	0.4	0.2	0.05	0.25

Calculate the Variance:

a) $Var(X) = 1.8475$

b) $Var(X) = 1.3592$

c) $Var(X) = 3.95$

d) $Var(X) = 13.5$

7. Solve the equation $\tan x = -\sqrt{3}$, for $0° \leq x \leq 360°$.

a) $x = -60°$

b) $x = 60°$ and $240°$

c) $x = 120°$

d) $x = 120°$ or $300°$

8. Find the acute angle between the lines $y = 3x - 4$ and $y = \frac{6}{5}x + 3$ correct to the nearest degree:

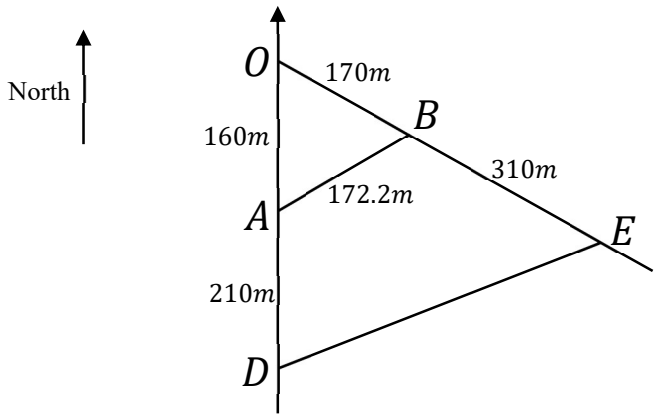
a) 72°

b) 50°

c) 21°

d) 22°

9. A farm was being measured for an upcoming orienteering mini exercise. The triangular field ODE is to be used for this activity. The lengths of the fences are marked on the diagram below where point O is due north of point D, $DA = 210m$, $AO = 160m$, $OB = 170m$, $BE = 310m$ and $AB = 172.2m$



Find the bearing of D from E.

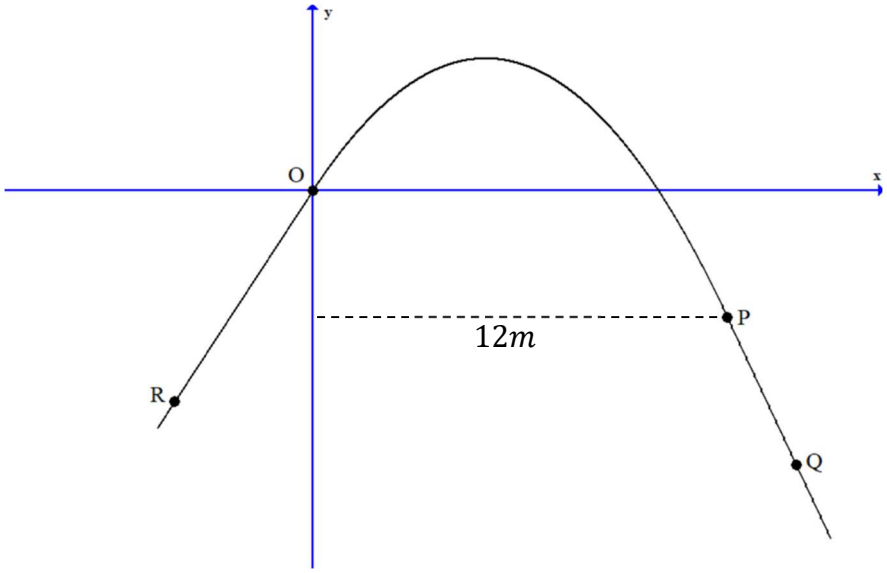
a) 071°

b) 251°

c) 047°

d) 313°

10. Luna Park is designing a new roller coaster, and the diagram below illustrates part of its design. The section RO is a straight line with $m = 2.5$ and the section PQ is a straight line with $m = -3.5$. The section OP is a parabola of the form $y = ax^2 + bx$. The horizontal distance from the y-axis to P is $12m$.



For the ride to be smooth, the straight-line sections must be tangents to the parabola at O and P.

What are the values of a and b that make the ride is smooth?

a) $a = -0.0625$ and $b = 2.5$

b) $a = -0.25$ and $b = 2.5$

c) $a = -2.5$ and $b = -3.5$

d) $a = -2$ and $b = 1$

2024 Gosford High School Year 11 Yearly

Student number

Mathematics Advanced

Section II Answer Booklet

Section II

70 marks

Attempt Questions 11–29

Allow about 1 hour and 45 minutes for this section.

Instructions

- **Write your Student Number at the top of this page.**
- **Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.**
- **Your responses should include relevant mathematical reasoning and/or calculations. Bald answers may not be awarded full marks.**
- **Extra writing space is provided on pages 17-18. If you use this space, clearly indicate which question you are answering.**

Question 11. (1 marks)

A single card is drawn from a regular deck of cards. What is the probability that the card is a red four?

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Question 12. (1 mark)

Evaluate $|6 - |7 \times -4| \times 3|$

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Question 13. (4 marks)

a) Explain, using exponentials, between which 2 consecutive integers the value of $\log_3 30$ must lie. 2 marks

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b) Then, by first expressing it in terms of $\log_{10}$, find the value of $\log_3 30$ correct to 2 decimal places. 2 marks

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Question 14. (8 marks)

Differentiate the following, simplifying fully where possible:

a) $f(x) = \frac{1}{x^2} + \sqrt{x}$ 2 marks

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b) $f(x) = 3x(4x^2 + 5)^4$ 3 marks

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c) $f(x) = \frac{x^2}{(x-4)^3}$ 3 marks

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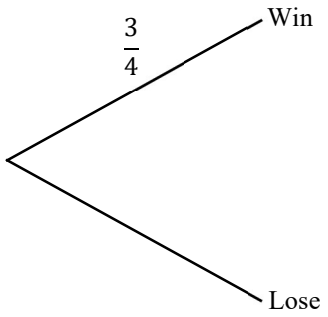
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Question 15 (5 marks)

Perry is a competitive tennis player. She is trying to win the Big Tennis Cup and to do so must win her quarter final, semi-final and final match. If she loses any game, she will not play any further games. In the quarter final, she is three times as likely to win as her opponent. If she makes it to the semi-final she is twice as likely to win as her opponent and if she makes it to the final, Perry and her opponent have equal chances of winning.

a) Complete the tree diagram below to represent this information. 2 marks

Quarter final



b) What is the probability that Perry wins the Big Tennis Cup? 1 mark

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c) What is the probability that Perry loses in the final, given that she wins her quarter final match? 2 marks

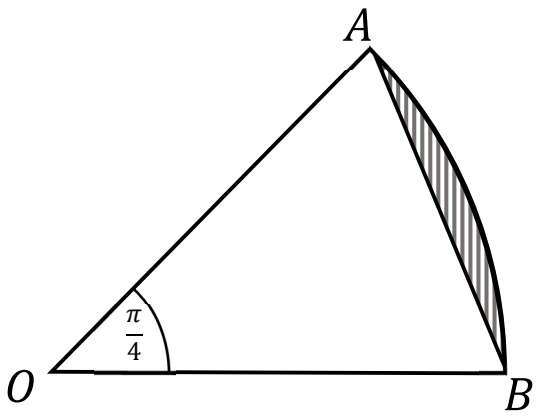
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Question 16. (3 marks)

In the diagram below OAB is a sector of a circle with centre O, radius $8cm$ and $\angle AOB = \frac{\pi}{4}$. Determine the exact area of the segment created by the chord AB (the shaded area).



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Question 17. (3 marks)

Using derivatives, find the equation of the tangent to the curve $y = 4x^3 - 3x^2 + x$ at the point $x = \frac{1}{2}$. Answer in general form.

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Question 18. (3 marks)

The lines $x + 2y - 8 = 0$, $y = x - 2$ and $y = 0$ enclose a triangular region. Find the area of the triangular region.

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Question 19. (2 marks)

Given that $x = \log_a 3$ and $y = \log_a 7$, write an expression for $\log_a \frac{27}{7}$ in terms of x and y.

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Question 20 (2 marks)

In a bag there are 3 six-sided dice. Two of the dice have faces marked 1, 2, 3, 4, 5, 6. The other is a special die with faces marked 1, 5, 5, 5, 5, 6.
One die is randomly selected and tossed.

a) What is the probability that the die shows a 5? 1 mark

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b) Given that the die shows a 5, what is the probability that it is the special die? 1 mark

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Question 21. (3 marks)

Simplify fully: $\dfrac{4(4x+1)}{3x^2+5x-2} - \dfrac{4}{x+2}$

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Question 22. (4 marks)

a) Show that $\sin\theta \cot\theta = \cos\theta$ 1 mark

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b) Hence, solve $8\cos\theta \sin\theta \cot\theta = \sec\theta$ for $0 \leq \theta \leq 2\pi$ 3 marks

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Question 23. (6 marks)

Gerry's displacement from home is modelled by the equation $x = -0.5t^2 + 2t + 6$ where x is measured in kilometres and t is measured in hours.

a) What is his initial displacement? 1 mark

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b) What is Gerry's velocity after 2 hours? 2 marks

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c) Sketch the graph of velocity vs time for Gerry's journey for $0 \leq t \leq 6$. 3 marks

Question 24. (3 marks)

Find the derivative of $f(x) = 11 - x^2$ through first principles given that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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Question 25 (3 marks)

Events A and B from a sample space, are such that $P(A)=\frac{1}{4}$ and $P(B)=\frac{2}{5}$.

a) Find $P(A'\cap B)$ if $P(A\cap B)=\frac{1}{6}$ 2 marks

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b) Find $P(A'\cap B)$ if A and B are mutually exclusive events. 1 mark

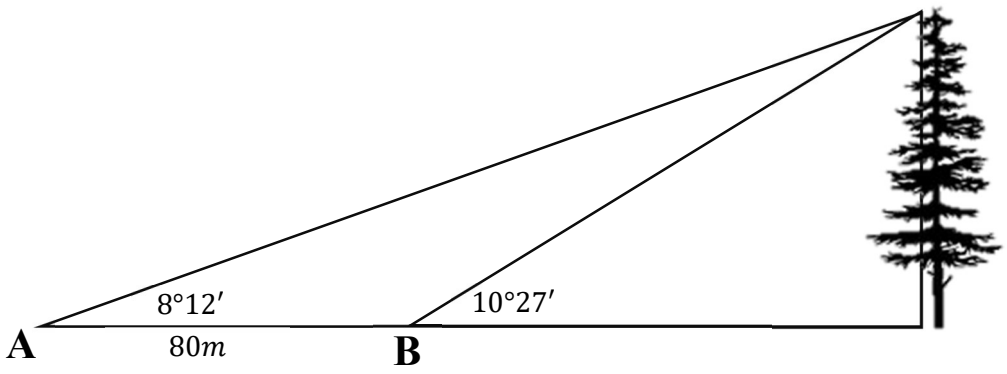
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Question 26 (4 marks)

An environmentalist is surveying the trees of a local park to determine their current heights. For one particular tree she begins at point A and uses her clinometer to determine the angle of elevation to the top of the tree to be $8°12'$. She then walks $80m$ towards the tree and determines the angle of elevation from her new position, B, to be $10°27'$. This information is shown in the diagram below.



Determine the height of the tree to 3 significant figures.

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Question 27 (3 marks)

Prove that $\sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta = \tan\theta + \cot\theta$

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Question 28 (6 marks)

The probability distribution of a discrete random variable, X, is given by the table below.

x	-1	0	1	2	3
$P(X=x)$	$2p^2$	$\frac{3}{8}p$	$1-2.5p$	p	$\frac{7}{32}-p^2$

a) Show that $p=\frac{1}{4}$. Explain why this is the only valid solution. 3 marks

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b) Calculate $E(X)$. Answer in exact form. 2 marks

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c) Find $P(X > E(X))$ 1 mark

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Question 29 (6 marks)

A parabola has the equation $y = Ax^2 + 1$ where A is an unknown constant and has a tangent with the equation $y = 10x + 11$.

a) Find the value of A. 2 marks

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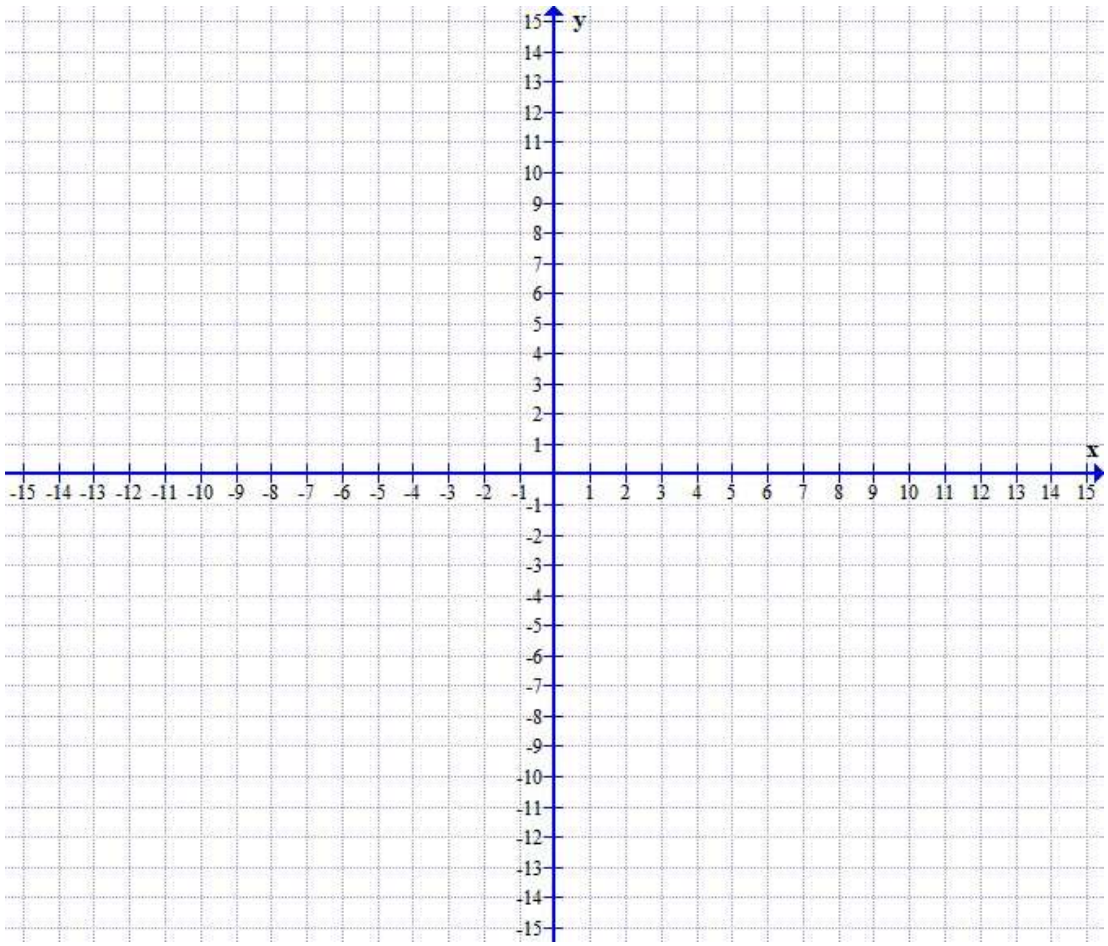
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b) Sketch the parabola and the tangent on the grid below, clearly labelling all key points including the point of contact between the parabola and it's tangent. 4 marks



END OF EXAM

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student number

2024 Year 11 Yearly Examination

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Section 1: Student responses

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○