
Student number

Gosford High School

2023 Year 11 Yearly Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- In Questions 11–28, show relevant mathematical reasoning and/ or calculations

Total Marks

70

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Detach the *Multiple-choice answer sheet* from the last page of this question booklet.

Section II – 60 marks

- Attempt Questions 11–28
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Question 1-10

1 Which of the following is equal to $\dfrac{x^2-3x-4}{x^2-16}$?

A. $\dfrac{x-1}{x+4}$

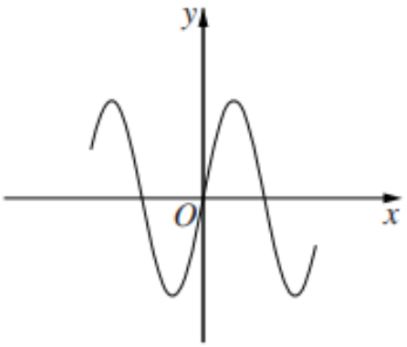
B. $\dfrac{x+1}{x-4}$

C. $\dfrac{x-1}{x-4}$

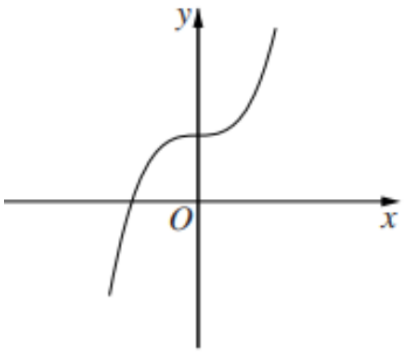
D. $\dfrac{x+1}{x+4}$

2 Which of the following graphs represent an odd function?

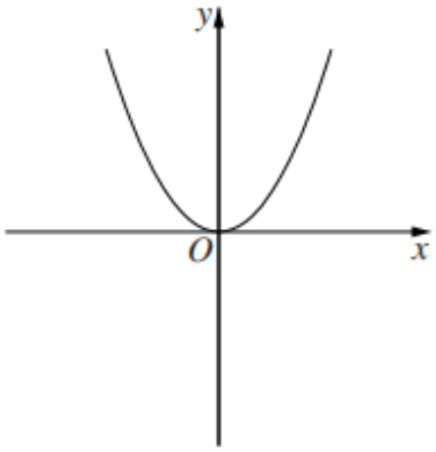
A.



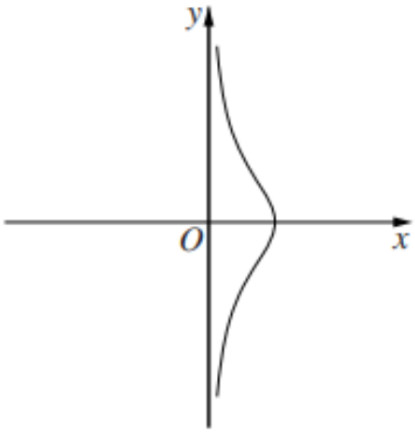
B.



C.



D.



3 Find the gradient between the points $(1,2)$ and $(-2,-7)$?

A. -3

B. 3

C. $-\frac{1}{3}$

D. $\frac{1}{3}$

4 If $\cos\theta = -\frac{1}{2}$ and $\pi \leq \theta < 2\pi$, what is the value of $\sin\theta$?

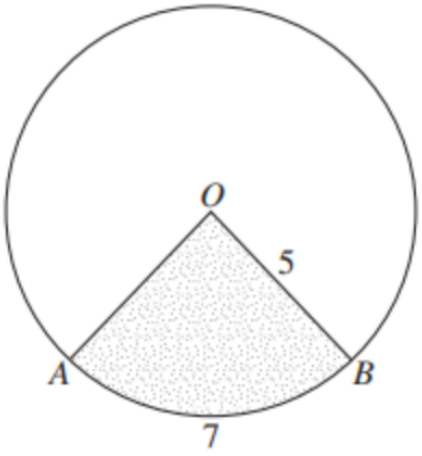
A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

5 The circle centres at O has radius 5. Arc AB has length 7 as shown in the diagram.



What is the area of the shaded sector AOB?

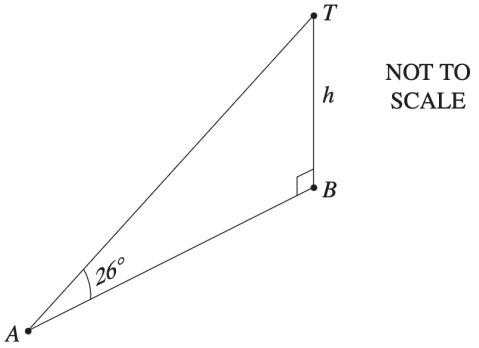
A. $\frac{125}{14}$

B. $\frac{35}{2}\pi$

C. $\frac{35}{2}$

D. $\frac{125}{14}\pi$

6 A tower BT has height h metres.
From point A, the angle of elevation to the top of the tower is $26°$ as shown.



Which of the following is the correct expression for the length of AB?

A. $h \cot 26°$

B. $h \tan 26°$

C. $h \sin 26°$

D. $h \operatorname{cosec} 26°$

7 What domain makes the function $f(x) = -2x^2 - 8x + 10$ one-to-one?

A. $(-\infty, -2]$

B. $(-\infty, 18]$

C. $(-\infty, \infty)$

D. $(-\infty, 2]$

8 If $A = \{3, 6, 9, 12, 15\}$ and $B = \{2, 4, 6, 8, 10, 12, 14\}$, which set is represented by $A \cup B$?

A. $\{\emptyset\}$

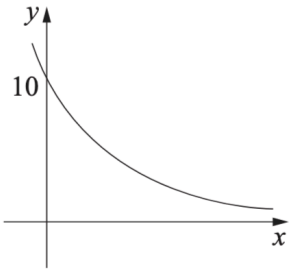
B. $\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15\}$

C. $\{2, 3, 4, 8, 10, 14, 15\}$

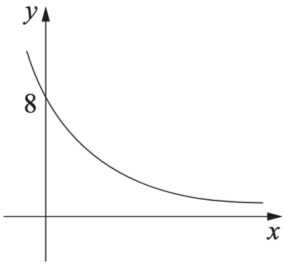
D. $\{6, 12\}$

9 Which of the following best represents $y = 10(1.8)^x$

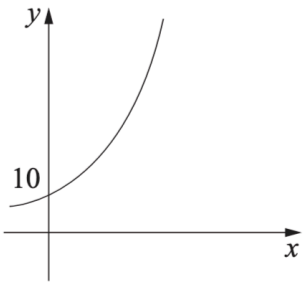
A.



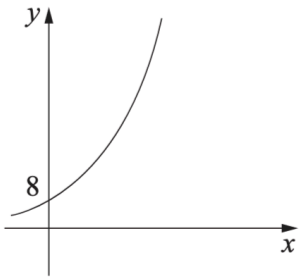
B.



C.



D.



10 Bag A contains three blue balls and four orange balls, while bag B contains five blue balls and two orange balls. A bag is chosen at random and then two balls are drawn from it without replacement.

What is the probability that both balls are orange?

A. $\frac{1}{21}$

B. $\frac{2}{7}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

- Blank Page -

2023 Gosford High School Year 11 Yearly

Student number

Mathematics Advanced

Section II Answer Booklet

Section II

60 marks

Attempt Questions 11–28

Allow about 1 hours and 45 minutes for this section.

Instructions

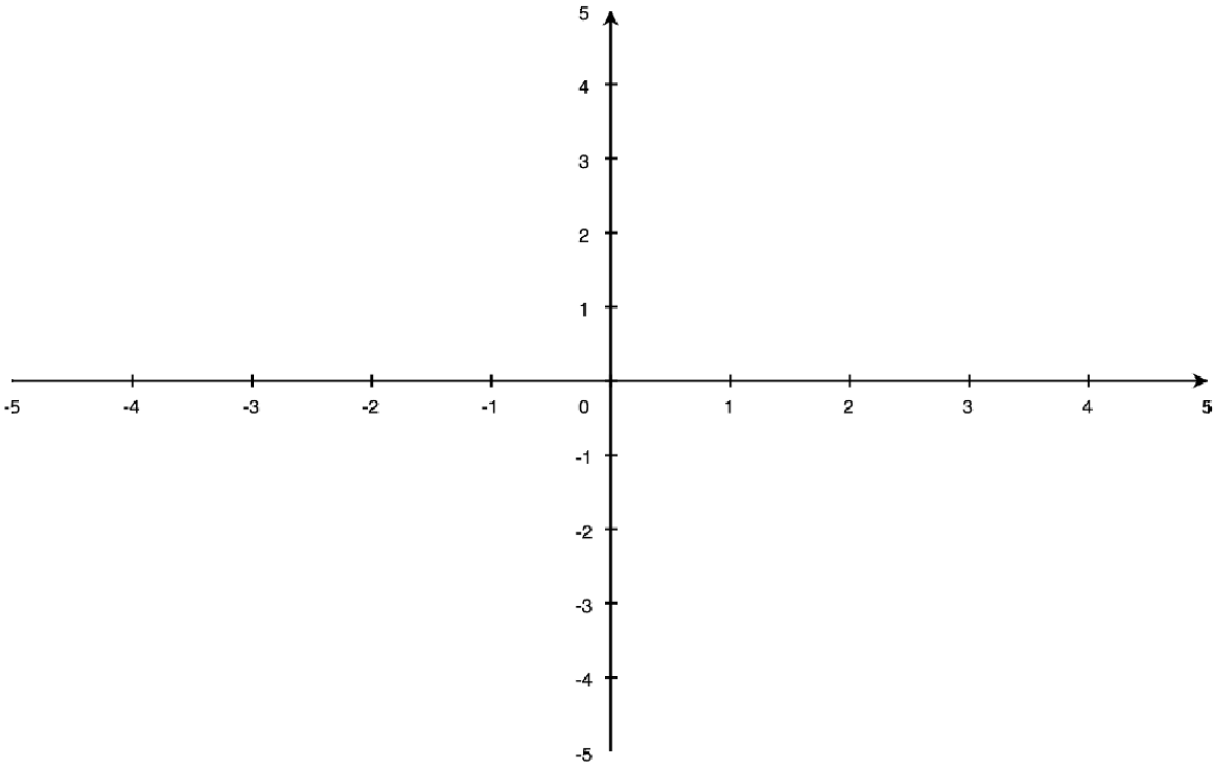
- **Write your Student Number at the top of this page.**
- **Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.**
- **Your responses should include relevant mathematical reasoning and/or calculations.**
- **Extra writing space is provided on pages 21-22. If you use this space, clearly indicate which question you are answering.**

Question 11 (2 marks)

Rationalise $\dfrac{2\sqrt{6}}{3\sqrt{5}}$.

Question 12 (2 marks)

(a) Sketch the graph of $y = |x - 2|$ **1**



(b) Hence or otherwise, solve $|x - 2| = 1$. **1**

Question 13 (5 marks)

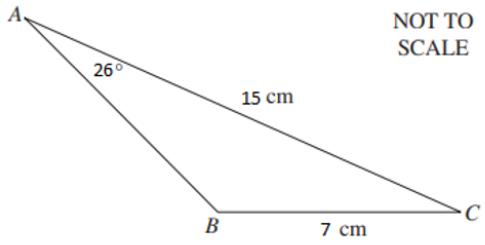
Differentiate the following:

(a) $f(x) = \sqrt[3]{x^2 + 6x - 3}$ **2**

(b) $f(x) = e^{2x}(2x + 1)$ **3**

Question 14 (3 marks)

Find the obtuse angle ABC in the triangle below:



Question 15 (3 marks)

Find the exact value of:

(a) $\tan 30°$ **1**

(b) $\cos 225°$ **2**

Question 16 (2 marks)

Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ if $f(x) = x^2 - x - 3$.

Question 17 (2 marks)

Prove that $\sec x - \cos x = \sin x \tan x$

Question 18 (6 marks)

Consider the functions $f(x) = \frac{1}{x}$ and $g(x) = x^2 - x - 6$.
Let $h(x) = f(g(x))$.

(a) State the rule for $h(x)$ **1**

(b) State the domain of $h(x)$ **1**

(c) Let $i(x) = g(f(x))$. For what value of x does $g(x) = i(x)$? **4**

Question 19 (3 marks)

Let $f(x) = 1 + e^{2x}$

Show that $f(x) \times f(-x) = f(x) + f(-x)$

Question 20 (4 marks)

A particle moves in a straight line described by the function $x(t) = -t^2 + 5t + 6$.

(a) Find the velocity as a function of time. **1**

(b) When does the particle change directions? **1**

(c) At what times is the particle 10 units away from the origin? **2**

Question 21 (7 marks)

A die is weighted so that the probability of rolling a 1 or 2 is 20% more likely to occur than rolling a 3, 4, 5, or 6. They are equally likely to occur in their own groups.

(a) Fill in the missing values in the table **1**

x	1	2	3	4	5	6
$P(X = x)$						

(b) Graph the distribution of this experiment. **1**



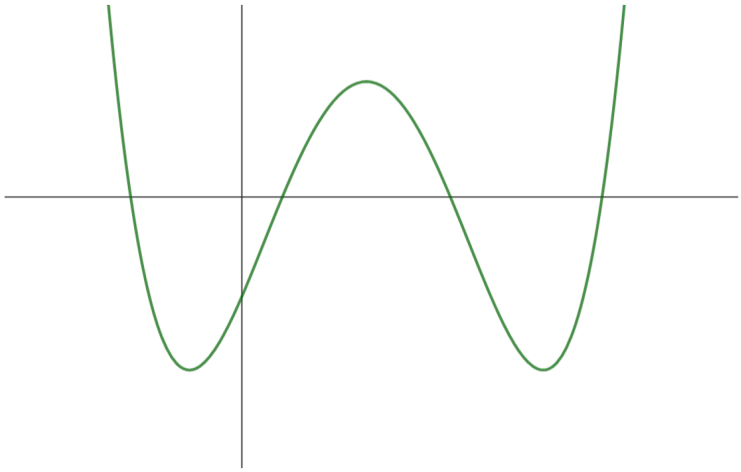
(c) Show that the expected value 2.7 **2**

(d) Show that the variance is 2.81 **2**

(e) Find the standard deviation. **1**

Question 22 (3 marks)

The diagram below shows the graph of $y = f(x)$



Sketch the graph of $y = f'(x)$ on the diagram above.

Question 23 (3 marks)
A plane starts from point A and flies on a bearing of $202°$ for 12 km and then on a bearing of $282°$ until it is due west of its starting position.

How far west is the plane from its starting position?

Question 24 (3 marks)

The velocity of a function is given by $v = \dfrac{2t}{9 + t^2}$.

Find an expression for the acceleration.

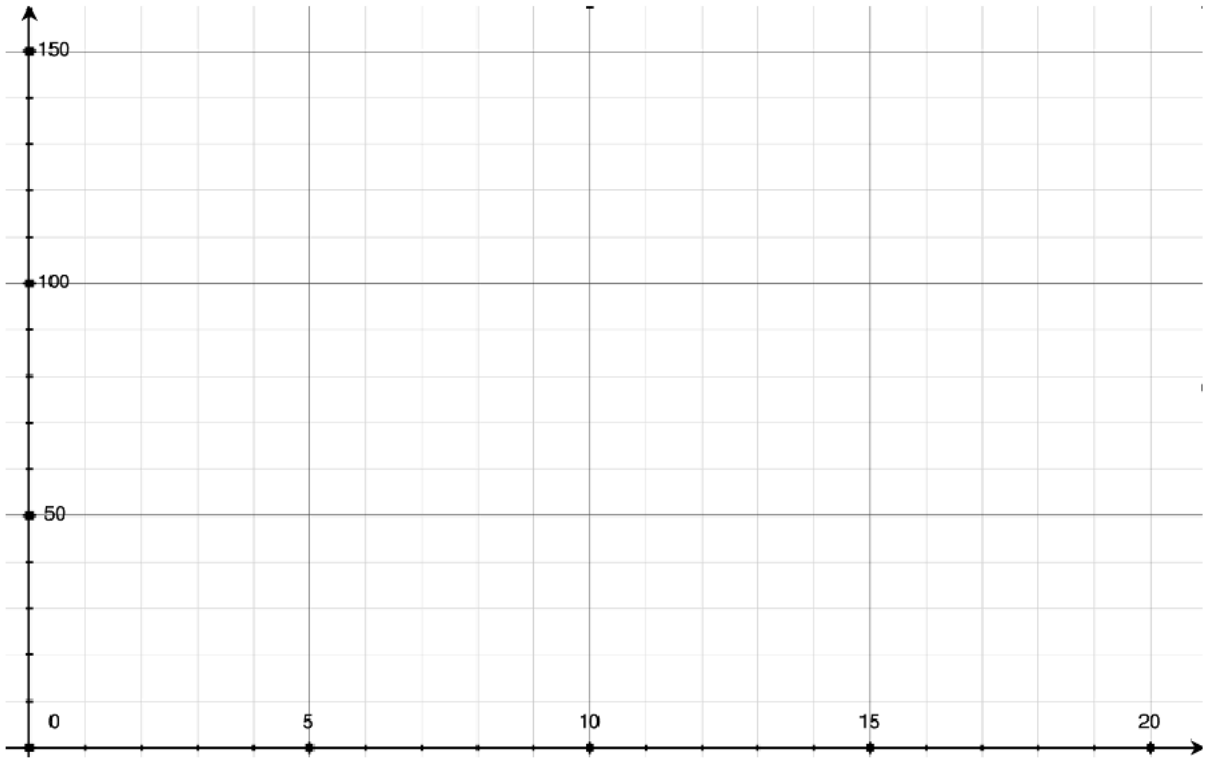
Question 25 (6 marks)

A metal bar is heated and then allowed to cool. It's temperature T in $°C$ is found to be $T = 15 + 125e^{-0.3t}$ where t (in minutes) is the time of cooling.

(a) Find the initial temperature of the metal bar. **2**

(b) How cool is the metal bar after 10 minutes? **1**

(c) Graph the temperature vs time for $0 \leq t \leq 20$. **2**



(d) Using the graph or otherwise, find the time when the temperature reaches $30°C$. **1**

Question 26 (5 marks)

Let $y = 3x - x^2$.

(a) Show that the gradient of the tangent at $x = 3$ is equal to -3. **2**

(b) Hence or otherwise, find the equation for the normal at $x = 3$. **3**

Question 27 (6 marks)

A survey of 70 students showed that 40 students studied chemistry, 25 students studied physics and 11 students studied both.

(a) Draw a Venn diagram below depicting this information. **2**

(b) What is the probability that a student chosen at random studies physics? **1**

(c) What is the probability that a student chosen at random studies chemistry but not physics? **1**

(d) What is the probability a student studies physics given that they also study chemistry? **2**

Question 28 (4 marks)

Consider the function defined by $f(x) = \begin{cases} ax^2 + x - b, & x > 2 \\ ax + b, & 2 \leq x \leq 5 \\ 2ax - 7, & x > 5 \end{cases}$. Given the function is continuous at $f(2)$ and $f(5)$, find the values of a and b.

END OF EXAM

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student number

2023 Year 11 Yearly Examination

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ● B ⊗ (correct) C ○ D ○

Section 1: Student responses

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○