10 marks

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ←

(A) ⊗ (B) ⊗ (C) ○ (D) ○

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1. What are the solutions to the equation $\sin x = \frac{\sqrt{3}}{2}$, for $0 \le x \le 2\pi$?

A. $\frac{\pi}{6}, \frac{5\pi}{6}$

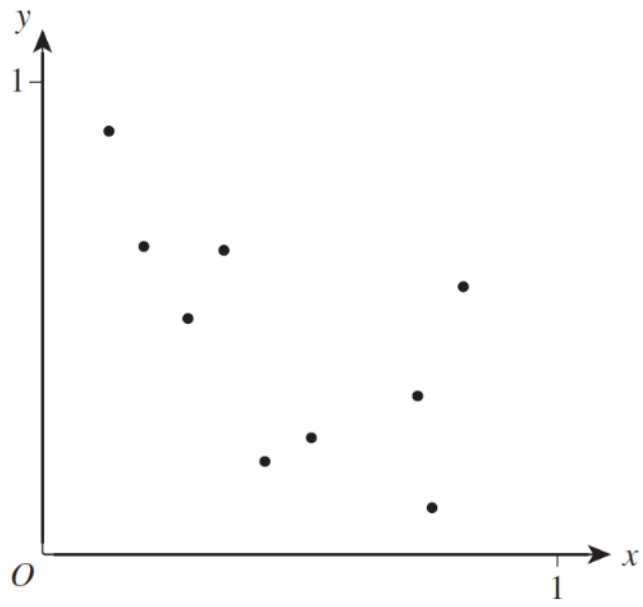
B. $\frac{\pi}{3}, \frac{2\pi}{3}$

C. $\frac{\pi}{4}, \frac{3\pi}{4}$

D. $\frac{\pi}{2}, \frac{3\pi}{2}$

2. A scatterplot relates the quantities x and y.

How could you describe the correlation between those quantities?



A. A moderate negative correlation

B. A moderate positive correlation

C. A weak positive correlation.

D. A strong negative correlation

3. For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > 6$

C. $x < -6$

D. $x > -\frac{1}{6}$

4. What is the period for the curve $y = -3\cos\left(2x - \frac{\pi}{4}\right)$?

A. 3

B. π

C. 2π

D. -3

5. Which one of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

A. $\left[\frac{1}{2}, 2\right]$

B. $\left(\frac{1}{2}, 2\right)$

C. $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

D. $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

6. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

A. $3x^3(4-x)^2(7x-16)$

B. $3x^3(4-x)^2(16-7x)$

C. $3x^3(4-x)^3(16-7x)$

D. $3x^3(4-x)^3(7x-16)$

7. The graph of $y = f(x)$ has a stationary point at $(2, -3)$.

Consequently, which of the following is a stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

A. $(4,\ 2)$

B. $(4, -2)$

C. $(1,\ 2)$

D. $(1, -2)$

8. For the series $2\pi, \pi, \frac{\pi}{2}, \ldots\ldots\ldots$, calculate the exact value of the sum of the first 6 terms.

A. $\frac{63\pi}{16}$

B. $\frac{7\pi}{2}$

C. $\frac{977\pi}{256}$

D. $\frac{63\pi}{64}$

9. Consider the region bounded by the x-axis, the y-axis, the line with equation $y = 3$ and the curve with equation $y = \ln(x - 1)$. The exact value of the area of this region is:

A. $e^{-3} - 1$

B. $e^3 + 2$

C. $3e^2$

D. $3e^3 - e^{-3} + 2$

10. A lie detector was used to indicate the guilt or innocence of 200 suspects.

	Accurate	Not accurate	Total
True statements	95	10	105
False statements	70	25	95
Total	165	35	200

What is the probability a person selected at random, with an accurate test, made a true statement?

A. $\frac{95}{105}$

B. $\frac{95}{200}$

C. $\frac{95}{165}$

D. $\frac{165}{200}$

Section II **Student Number:**

90 marks

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

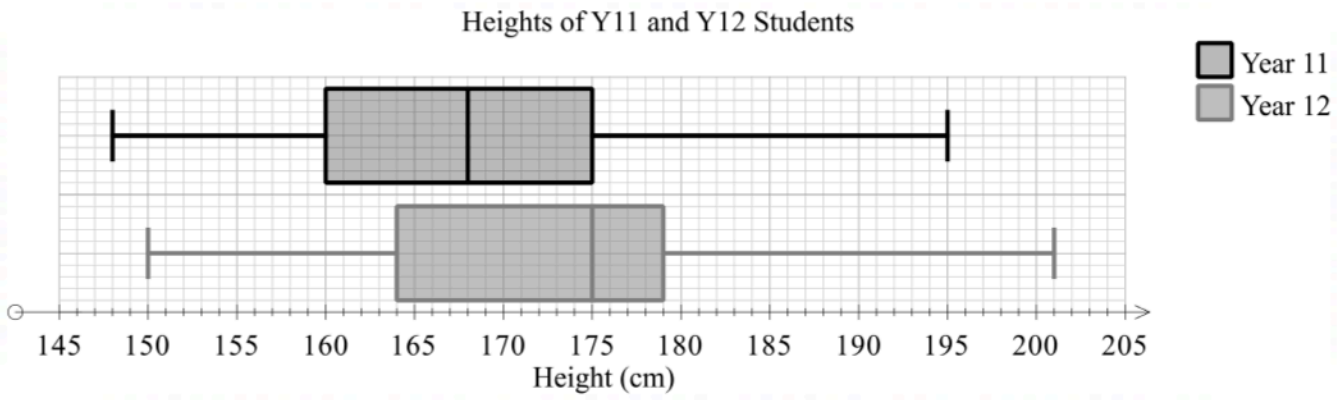
Question 11 (15 marks) **Marks**

(a) Find a primitive of $\frac{1}{5x+1}$. **2**

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(b) The boxplot shows the heights of students in Year 11 and Year 12 at a school.



(i) What percentage of students in Year 11 have a height below 160 cm? **1**

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(ii) The number of students taller than 175 cm is coincidentally the same for both Year 11 and Year 12. If Year 11 has a total of 140 students, how many students are in Year 12? **1**

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Question 11 continues on page 10

(c) Find the common ratio of a geometric series with a first term of $\sqrt{2}$ and a limiting sum of $\frac{3\sqrt{2}}{2}$. **2**

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(d) Find $\int \frac{8x^3 - 3}{x^2}\, dx$ **2**

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Question 11 continues on page 11

(e) A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(i) Find $\frac{d^2y}{dx^2}$ **1**

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(ii) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. **2**

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(iii) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

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Question 11(e) continues on page 12

(iv) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

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End of Question 11

Question 12 (15 marks) **Student Number:**

(a) Let $h(x) = (x-2)(x^2+1)$.

(i) Find where the graph of $y = h(x)$ cuts the x-axis and y-axis. **2**

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(ii) Find the coordinates of the stationary points on the curve with the equation $y = h(x)$ and determine their nature. **5**

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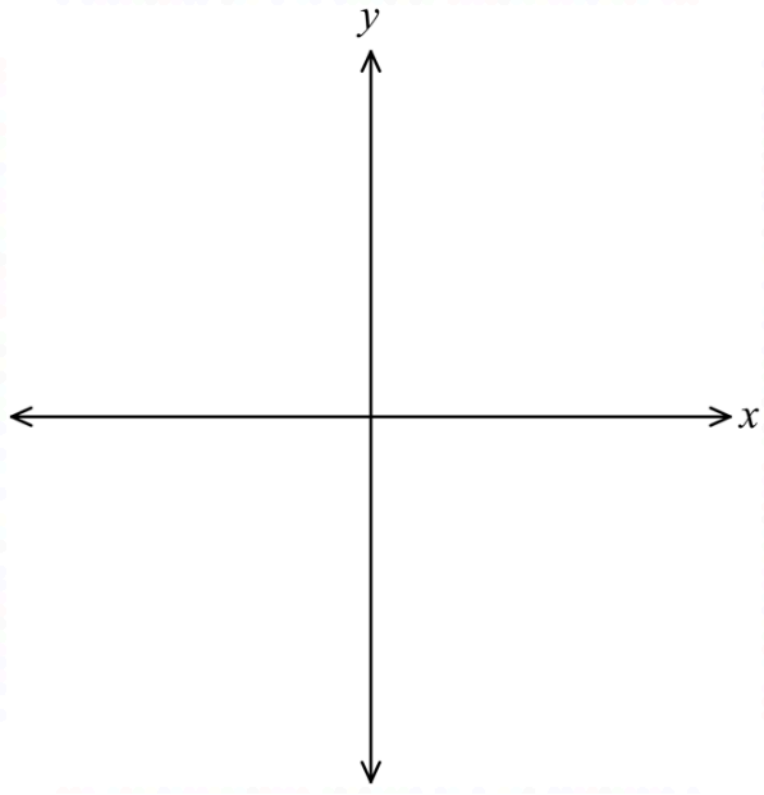
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Question 12 continues on page 16

(iii) Sketch the graph of $y = h(x)$ showing stationary points and axes intercepts. **2**



(b) If $\tan\theta = \frac{4}{5}$, and θ is acute, find the exact value of $\sin\theta$. **2**

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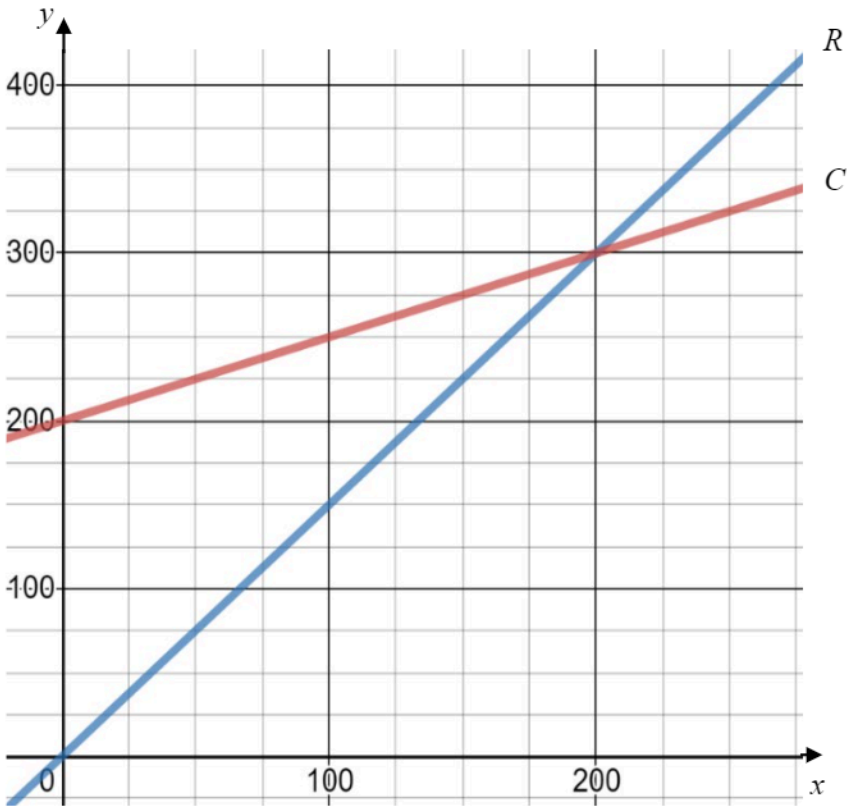
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Question 12 continues on page 17

(c) Terry is starting a small business making face masks.

Technology was used to draw straight-line graphs to represent the cost C, of Terry making face masks and the revenue, R from him selling them. The x-axis displays the number of face masks and the y-axis displays the cost/revenue in dollars.



(i) How many face masks must Terry sell to break even? **1**

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Question 12(c) continues on page 18

(ii) By first forming equations for cost C, and revenue R, determine how many face masks need to be sold to earn Terry a profit of \$1500. **3**

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End of Question 12

Question 13 (15 marks) **Student Number:**

(a) Find the exact value of $\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx$ **2**

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(b) Differentiate with respect to x.

(i) $y = \ln\left(3x^2 + 1\right)$ **1**

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(ii) $y = \dfrac{\sin x}{x^2}$ **2**

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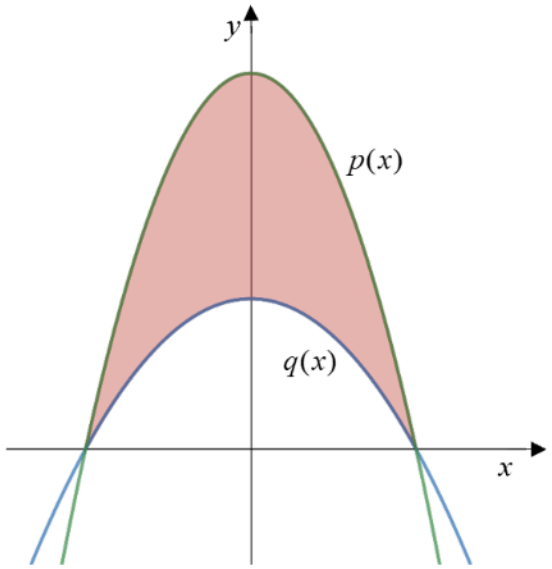
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Question 13 continues on page 22

(c) Tess is creating a logo from the region intersecting the curves:

$$p(x) = (5-x)(5+x) \quad \text{and} \quad q(x) = \frac{2}{5}(5-x)(5+x).$$



(i) Show that the area A, of the shaded region is given by the expression **3**

$$A = \frac{6}{5}\int_0^5 25 - x^2 \, dx.$$

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Question 13(c) continues on page 23

(ii) Hence, or otherwise, find the area of the shaded region. **2**

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(d) For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{7}$. **1**

Calculate $P(A \cap B)$.

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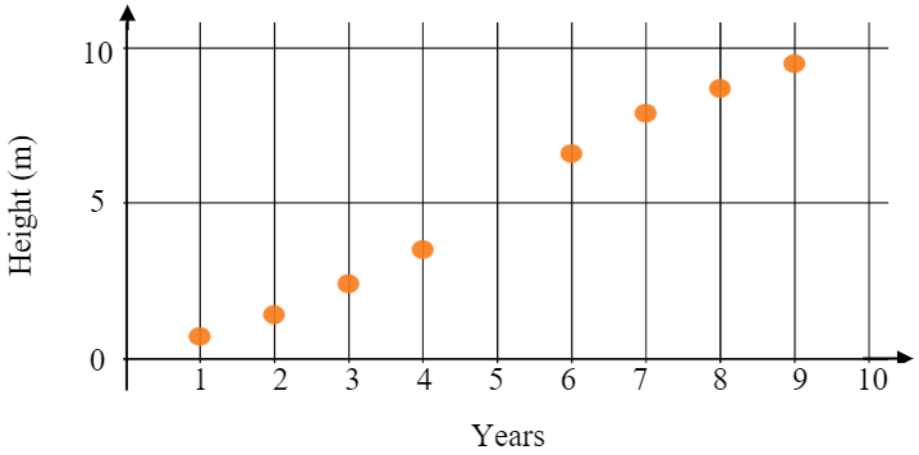
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Question 13 continues on page 24

(e) Charlotte is an agricultural scientist studying the growth of a particular tree over several years. The data she recorded is shown in the table and graph below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5



(i) What is the correlation coefficient for this data (correct to 4 decimal places)? **1**

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(ii) Find the equation of the least-squares line of best fit in terms of years (t) and height (H). Answer using values A and B correct to 2 decimal places, where $H = A + Bt$. **1**

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(iii) Use the equation to approximately determine how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

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(iv) What is the limitation of this model? **1**

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End of Question 13

Question 14 (15 marks) **Student Number:**

(a) The probability that Chloe gets a concert booking with her band on any given weekend is 65%. What is the probability that she gets at least one booking over two consecutive weekends? **2**

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(b) A circle is given by the equation $x^2 + y^2 - 4x + 6y = 12$.

Find the centre and radius of this circle. **3**

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Question 14 continues on page 28

(c) The score, X, for a biased spinner is given by the probability distribution:

x	2	4	6
$P(X = x)$	$\frac{1}{12}$	$\frac{2}{3}$	p

By finding the value of p , calculate the expected value and the variance of X. **3**

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Question 14 continues on page 29

(d) The displacement of a particle is given by $x = t^2 - 4\log_e(t-1) + 5$,

where x is in metres, t is in seconds and $t > 1$.

(i) Find the exact displacement of the particle when $t = 4$. **1**

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(ii) Find an expression for the particle's velocity and hence find when the particle comes to rest. **2**

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(iii) Show that the acceleration remains positive for $t > 1$. **2**

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Question 14(d) continues on page 30

(iv) Find the exact distance travelled by the particle between the times the particle comes to rest and $t = 4$. **2**

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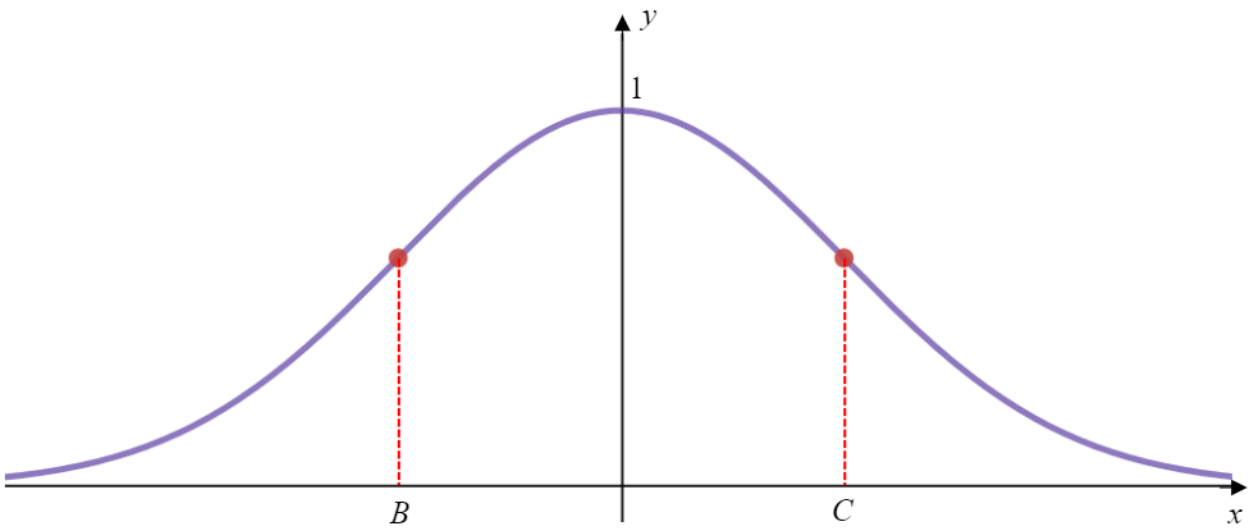
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End of Question 14

Question 15 (15 marks) **Student Number:**

(a) Isabelle is exploring the curve of the even function shown below, $y = e^{-x^2}$.
She knows there is a single stationary point shown at (0, 1) and two points of inflection are shown with x-values of B and C.



(i) Show that $\frac{d^2y}{dx^2} = 4e^{-x^2}\left(x^2 - \frac{1}{2}\right)$ **2**

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Question 15(a) continues on page 34

(ii) Hence find the coordinates of the two points of inflection. **2**

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Isabelle wants to find the area of the region under the curve $y = e^{-x^2}$ bounded by the x-axis and the two inflection points, that is, $\int_B^C e^{-x^2}\,dx$.

(iii) Explain how using a formula given on the Reference Sheet is unable to help provide an answer. **1**

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Question 15(a) continues on page 35

(iv) Isabelle decides to approximate the area using the Trapezoidal Rule.

Show how she determined $\int_B^C e^{-x^2}\,dx \approx \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2e}}$ using three function values. **3**

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(v) Explain why Isabelle correctly knows $\int_B^C e^{-x^2}\,dx > \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2e}}$. **1**

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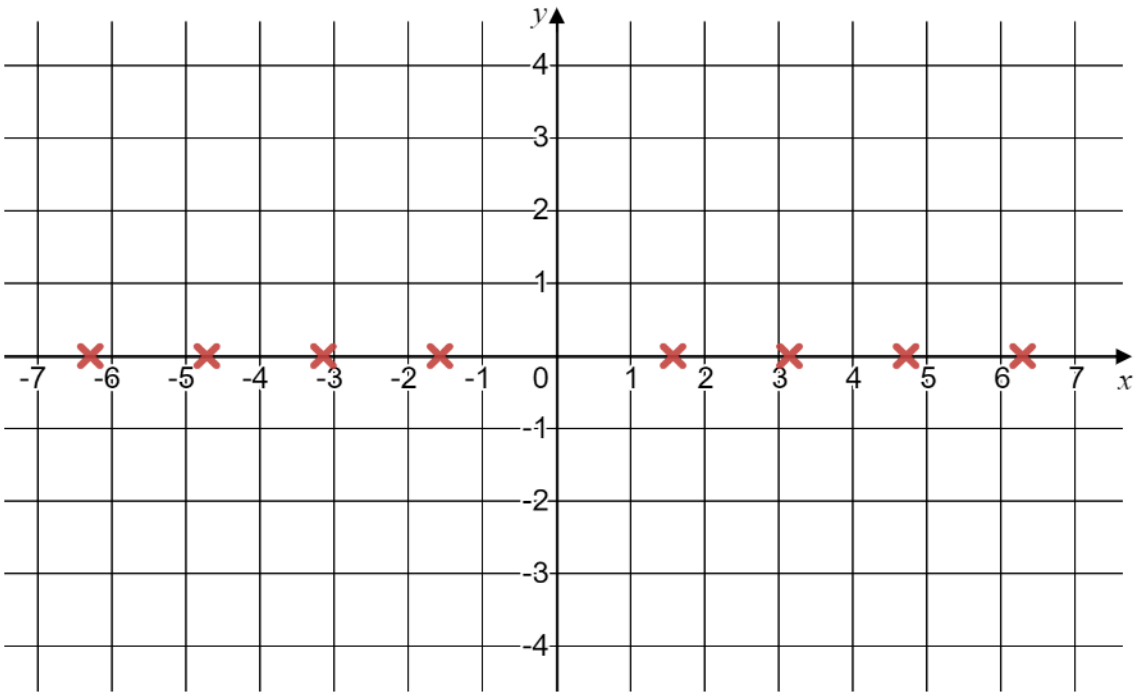
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Question 15 continues on page 36

(b) Annie was preparing to determine how many solutions there are to the equation:

$$3\sin x = \frac{x^2}{5} - 3.$$

She plotted multiples of $\frac{\pi}{2}$ on the x-axis of the number plane below, shown by the crosses, to help. Draw graphs on this number plane to solve Annie's problem. **3**



The number of solutions: _____

Question 15 continues on page 37

(c) Show that $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$ **3**

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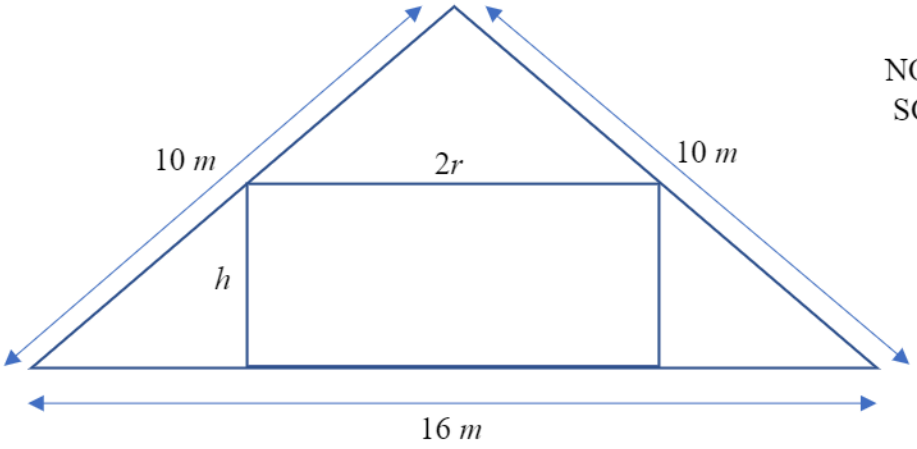
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End of Question 15

Question 16 (15 marks) **Student Number:**

(a) In some rural areas hot water tanks are installed in the roofs of houses. The diagram below shows a cross-section of a cylindrical tank in a roof. The cylindrical tank snugly fits exactly into the roof with diameter $2r$ metres and height h metres.
The cross-section of the roof is an isosceles triangle with dimensions show.



(i) Show that the height of the roof is 6 metres. **1**

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(ii) Hence show that $h = \frac{3}{4}(8-r)$. **2**

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Question 16(a) continues on page 40

(iii) Show that the volume of the cylindrical tank can be expressed by **1**

$$V = \frac{3\pi}{4}\left(8r^2 - r^3\right).$$

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(iv) Find the value of r which gives the tank its greatest volume and calculate that volume, correct to the nearest litre. **4**

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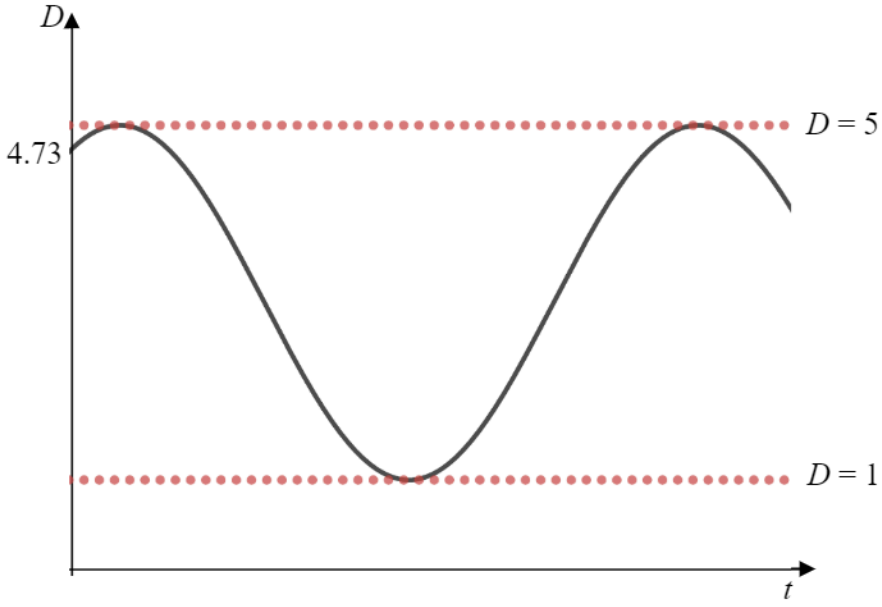
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Question 16 continues on page 41

(b)



Sophie has developed an equation, drawn above, for the depth D, of a river near her home. The depth is modelled by the function:

$$D = a\sin\left(nt + \frac{\pi}{3}\right) + c$$

where D is measured in metres and t is the time in hours. The time between successive peaks in Sophie's model is exactly 12 hours.

(i) What is the value of the amplitude, a? **1**

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(ii) Find the value of c. **1**

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Question 16(b) continues on page 42

(iii) Find the value of n. **1**

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Sophie started recording the river depth when it was 4.73 metres and waited to cross it safely.

(iv) How long did she have to wait until it had a depth of 1 metre? **2**

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. **Question 16(b) continues on page 43**

(v) From her record, when was the greatest rate of drop in depth and what was that rate at this time? Answer correct to 2 decimal places **2**

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END OF PAPER